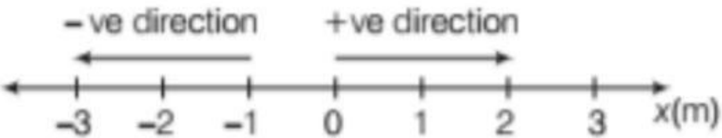
CHAPTER CHECKLIST

- Rest
- Motion
- Point Object
- Scalar and Vector Quantities
- Position, Path Length and Displacement
- Velocity
- Acceleration

- Acceleration
- Position-Time Graphs for Positive, Negative and Zero Acceleration
- Velocity-Time Graphs for an Accelerated Motion
- Kinematic Equations for Uniformly Accelerated Motion

Position

It is defined as the point where an object is situated. Position can be determined by the coordinate axis that is, marked in units of length and that has positive and negative directions.



Position in +ve and –ve direction

Frame of Reference

So, the frame of reference is a coordinate system with a clock w.r.t. which, an observer can describe the position, displacement, acceleration of an object.

Frame of reference are of two types

(i) **Inertial frame of reference** These are the frame of reference in which, Newton's first law of motion is applicable.

(ii) **Non-inertial frame of reference** These are the frame of reference in which, Newton's first law of motion is not applicable.

Accelerated and Unaccelerated Frames

- Generally, accelerated frames (with respect to earth) are non-inertial and unaccelerated frames (with respect to earth) are inertial.
- The frames at rest or in uniform motion are inertial and frames in non-uniform motion are non-inertial.
- To apply Newton's second law in the non-inertial frames, we use the concept of pseudo force.

Displacement

The change in position of an object in a particular direction is termed as **displacement**, i.e. the difference between the final and initial positions of the object. It is denoted by Δx. Mathematically, it is represented by

$$\boxed{\text{Displacement, } \Delta x = x_2 - x_1}$$

Cases

(i) If $x_2 > x_1$, then Δx is positive.
(ii) If $x_1 > x_2$, then Δx is negative.
(iii) If $x_1 = x_2$, then Δx is zero.

i.e. **the displacement of an object in motion can be positive, negative or zero.**

Displacement is a **vector quantity** as it possesses both, the magnitude and direction.

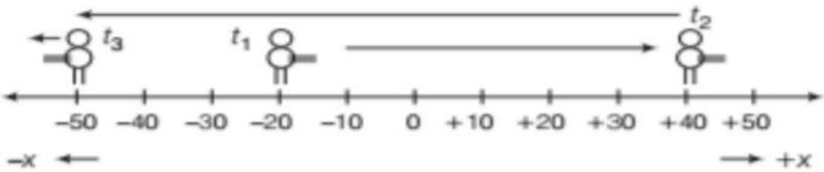
Differences between Distance and Displacement

S.No.	Distance	Displacement
1.	Length of the path is distance.	Magnitude of displacement is the length of the shortest possible path between initial and final positions.
2.	It is a scalar quantity.	It is a vector quantity.
3.	The distance covered by an object for some time interval cannot be zero.	The displacement of an object can be zero.
4.	The distance travelled by an object is always positive.	The displacement of an object can be positive, negative and zero

EXAMPLE |3| Motion of a Boy

A boy starts moving from –20 m towards + x-axis as shown in figure. He turns at time instant t_2 and starts moving towards – x-axis. At time t_3, he reached at –50 m as shown in the figure.

Find the displacement and distance for the time interval (i) t_1 to t_2 and (ii) t_1 to t_3.



For distance, we observe the actual path length and for displacement we observe the change in position.

Sol. (i) For t_1 to t_2,

Distance covered $= 20 + 40 = 60$ m

$$\text{Displacement} = 40 - (-20)$$
$$= +60 \text{ m (towards } + x\text{-axis)}$$

(ii) For t_1 to t_3,

Distance covered $= 60 + 90 = 150$ m

$$\text{Displacement} = -50 - (-20)$$
$$= -30 \text{ m (towards } - x\text{-axis)}$$

UNIFORM MOTION IN A STRAIGHT LINE

A body is said to be in a uniform motion if it travels equal distance in equal intervals of time along a straight line.

Note

For a uniform motion along a straight line in a given direction, the magnitude of the displacement is equal to the actual distance covered by the object.

Non-uniform Motion

A body is in non-uniform motion if it travels equal displacement in unequal intervals of time.

SPEED AND VELOCITY

Speed

The distance covered by an object divided by the time taken by the object to cover that distance is called the **speed** of that object.

$$\text{Speed} = \frac{\text{Distance travelled}}{\text{Time taken}}$$

Speed is a **scalar quantity**. The speed of the object for a given interval of time is always positive.

Unit of speed The unit of speed in MKS (SI) is m/s and in CGS as cm/s.

Dimensional formula $[M^0LT^{-1}]$.

Uniform Speed

If an object is moving with a uniform speed, it means that it covers equal distance in equal interval of time.

Variable or Non-Uniform Speed

If an object is moving with a non-uniform speed, it means that it covers equal distances in unequal intervals of time.

Velocity

The rate of change of position of an object with time is called the **velocity** of that object.

i.e. $$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$

Velocity is a **vector** quantity.

Unit of velocity The unit of velocity is cms^{-1} in CGS and ms^{-1} in MKS or SI.

Dimensional formula The dimensional formula of velocity is $[M^0LT^{-1}]$.

Uniform Velocity

An object could have uniform velocity if it covers equal displacement in equal interval of time. If an object have equal displacement in equal interval of time, it means that it

Non-uniform Velocity

If an object is moving with a non-uniform velocity, then it will have unequal displacements in equal interval of time.

Note The velocity of an object can be positive, zero and negative according to its displacement is positive, zero and negative.

Average Speed and Average Velocity

Average Speed

Average speed of an object is defined as the total distance travelled by the object divided by the total time taken.

$$\text{Average speed, } v_{av} = \frac{\text{Total distance travelled}}{\text{Total time taken}}$$

Average Velocity

Average velocity of a body is defined as the change in position or displacement (Δx) divided by the time interval (Δt) in which that displacement occur.

. Average velocity of the body is given by

$$\mathbf{v}_{av} = \frac{\Delta x}{\Delta t}$$

Average Velocity *vs* Average Speed

- For average speed, find net distance covered and divide it by time taken.
- Average velocity is net displacement divided by time taken.
- So, just find out the net displacement and divide it by time taken for that displacement.
- Average velocity could be zero or positive or negative but average speed is always positive for a moving body.

INSTANTANEOUS SPEED

Mathematically, instantaneous speed at any instant of time (t) is expressed as

$$\text{Instantaneous speed, } s_i = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} \text{ or } s_i = \frac{ds}{dt}$$

[ds is the distance covered in time dt]

where, $\frac{ds}{dt}$ is the differential coefficient of s w.r.t. t.

EXAMPLE |4| Instantaneous Speed of the Particle

If the average speed of the particle is $[2t^2 \hat{\mathbf{i}} + 3t \hat{\mathbf{j}}]$, then find out the instantaneous speed of the particle.

Sol. Given, position of the particle, $s = [2t^2\hat{\mathbf{i}} + 3t\hat{\mathbf{j}}]$

$$s_i = \frac{ds}{dt} = \frac{d}{dt}[2t^2\hat{\mathbf{i}} + 3t\hat{\mathbf{j}}]$$

Instantaneous speed of the particle is $s_i = 4t\hat{\mathbf{i}} + 3\hat{\mathbf{j}}$

INSTANTANEOUS VELOCITY

Mathematically, instantaneous velocity at instant of time (t) is given by

$$v_i = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \text{ or } v_i = \frac{dx}{dt}$$

EXAMPLE | 5| Instantaneous velocity of a particle
The displacement (in m) of a particle moving along x-axis is given by $x = 18t + 15t^2$. Find the instantaneous velocity at $t = 0$ and $t = 2$s.

Sol. Given, Displacement,

$$x = 18t + 15t^2$$

Instantaneous velocity,

$$v_i = \frac{dx}{dt} = 18 + 30t$$

Instantaneous velocity at

$t = 0$, $v = 18 + 30 \times 0 = 18$ m/s
$t = 2$s, $v = 18 + 30 \times 2 = 78$ m/s

Uniformly Accelerated Motion

ACCELERATION

The existence of acceleration was given by Galileo in his different thoughts. **Acceleration of a body can be expressed as the rate of change of velocity with time.** By acceleration, we can understand that how fast or slow the velocity of an object is changing. Acceleration is a **vector quantity.**

$$\text{Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

Its SI unit is metre per second square (m/s^2) and CGS is cm/s^2. The dimensional formula is $[M^0L^1T^{-2}]$.

Note

- Acceleration therefore, may result from a change in speed (magnitude), a change in direction or changes in both.
- If the signs of velocity and acceleration is same (both positive or both negative), the body will accelerate and when the signs of velocity and acceleration are opposite, it means that the body is retarding.

EXAMPLE |1| Retarding Bus

Starting from a stationary position, a bus attains a velocity of 6 m/s in 30 s. Then, the driver of the bus applies a brake such that the velocity of the bus comes down to 4 m/s in the next 5 s. Calculate the acceleration of the bus in both the cases.

Sol. **Case I** Initial velocity of the bus, $u = 0$

Final velocity, $v = 6$ m/s; Time taken, $t = 30$ s

$$\therefore \text{ Acceleration} = \frac{\text{Change in velocity}}{\text{Time taken}}$$

$$= \frac{\text{Final velocity} - \text{Initial velocity}}{\text{Time taken}} = \frac{v-u}{t} = \frac{6-0}{30}$$

$$= 0.2\,\text{m/s}^2$$

Case II Initial velocity, $u = 6$ m/s

Final velocity, $v = 4$ m/s

$$\therefore \quad \text{Acceleration} = \frac{v-u}{t} = \frac{4-6}{5} = -0.4\ \text{m/s}^2$$

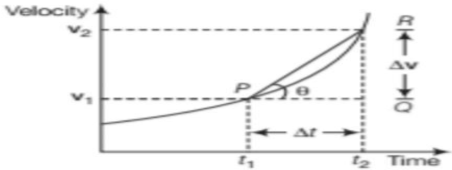
Thus, the acceleration in both the cases are $0.2\ \text{m/s}^2$ and $-0.4\ \text{m/s}^2$

Average Acceleration

The average acceleration over a time interval is defined as **the change in velocity divided by the time interval.**

Suppose that at any time (t_1) a body has velocity ($\mathbf{v}_1$). At a later time (t_2), it has velocity ($\mathbf{v}_2$). Thus,

Change in velocity, $\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1$
Time during which velocity has changed, $\Delta t = t_2 - t_1$



Velocity-time graph to calculate average acceleration

$$\mathbf{a}_{av} = \text{slope of straight line } PR = \frac{RQ}{PQ}$$

$$\text{Average acceleration, } \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{\mathbf{v}_2 - \mathbf{v}_1}{t_2 - t_1}$$

The slope of velocity-time graph gives acceleration.

Instantaneous Acceleration

It is defined as the acceleration of a body at a certain instant or the limiting value of average acceleration when time interval becomes very small or tends to zero.

So, $\boxed{\text{Instantaneous acceleration, } \mathbf{a}_{\text{inst}} = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{v}}{\Delta t} = \frac{d\mathbf{v}}{dt}}$

where $\frac{d\mathbf{v}}{dt}$ is the differential coefficient of $\mathbf{v}$ w.r.t. t.

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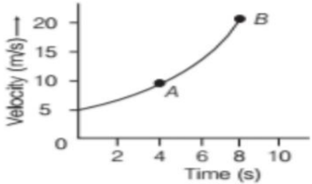
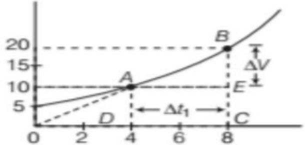
EXAMPLE |2| Calculation of Average Acceleration

From the figure given, find the average acceleration between points A and B.



Sol. We draw the graph as



Average acceleration, $\mathbf{a}_{av}$ = slope of line $AB = \frac{BE}{AE}$

$$BE = \Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_1 = 20 - 10 = 10 \text{ m/s}$$

$$AE = \Delta t = t_2 - t_1 = 8 - 4 = 4 \text{ s}$$

$$\therefore \quad \mathbf{a}_{av} = \frac{10}{4} = 2.5 \text{ m/s}^2$$

EXAMPLE |4| Average and Instantaneous Acceleration

A particle is moving in a straight line. Its displacement at any instant t is given by $x = 10t + 15t^3$, where x is in metres and t is in seconds. Find

(i) the average acceleration in the interval $t = 0$ to $t = 2$ s and

(ii) instantaneous acceleration at $t = 2$ s.

Sol. Given equation, $x = 10t + 15t^3$

and the variables are (i) $t = 0$ to $t = 2$s (ii) $t = 2$ s

Velocity of particle, $\mathbf{v} = \frac{dx}{dt}$

$$\mathbf{v} = \frac{d}{dt}(10t + 15t^3) = 10 + 45t^2$$

At $t = 0$, $\mathbf{v}_0 = 10 + 45(0) = 10$ m/s,

At $t = 2$ s, $\mathbf{v}_2 = 10 + 45 \times (2)^2$

$$= 10 + 180 = 190 \text{ m/s}$$

$$\Delta \mathbf{v} = \mathbf{v}_2 - \mathbf{v}_0 = 190 - 10 = 180 \text{ m/s}$$

$$\Delta t = 2 - 0 = 2 \text{ s}$$

$$\therefore \quad \mathbf{a}_{av} = \frac{\Delta \mathbf{v}}{\Delta t} = \frac{180}{2} = 90 \text{ m/s}^2$$

$$\mathbf{a} = \frac{d}{dt}(10 + 45t^2) = 90t$$

At $t = 2$ s, $\mathbf{a} = 90 \times 2 = 180$ m/s^2

The instantaneous acceleration of a particle at

Uniform Acceleration

If an object is moving with uniform acceleration, it means that the change in velocity is equal for equal intervals of time.

Non-uniform Acceleration

If an object has variable or non-uniform acceleration, it means that the change in velocity is unequal in equal intervals of time.

EXAMPLE |5| Uniform Acceleration

The displacement x of a particle varies with time t as $x = 4t^2 - 15t + 25$.

(i) Find the position, velocity and acceleration of the particle at $t = 0$.

(ii) Can we call the motion of the particle as one with uniform acceleration?

Sol. (i) Given position, $x = 4t^2 - 15t + 25$

Velocity, $v = \frac{dx}{dt} = \frac{d}{dt}(4t^2 - 15t + 25) = 8t - 15$

Acceleration, $a = \frac{dv}{dt} = \frac{d}{dt}(8t - 15) = 8$

At time $t = 0$, we have

$$x = 4t^2 - 15t + 25$$
$$= 4(0) - 15(0) + 25 = 25\,\text{m}$$
$$v = 8t - 15 = 8(0) - 15$$
$$= -15\,\text{m/s} \text{ and } a = 8\,\text{m/s}^2$$

(ii) Yes, the particle has a uniform acceleration because it does not depend on time t.

- **PROVE THREE EQUATIONS OF MOTION**