Dimension of physical quantity

The dimensions of a physical quantity are the powers (or exponents) to which the units of base quantities are raised to represent a derived unit of that quantity. There are seven base quantities and are represented with square brackets [ ] such as length [L], mass [M], time [T], electric current [A], thermodynamic temperature [K], luminous intensity [cd] and amount of substances [mol].

e.g. The volume occupied by an object is expressed as the product of length. So, its dimension is given by

$$V = [L] \times [L] \times [L] = [L^3]$$

As there is no mass and time in volume, so the dimension of volume is expressed as

$$V = [M^0 L^3 T^0]$$

Similarly, for force, it is the product of mass and acceleration. It can be expressed as

$$F = \text{mass} \times \text{acceleration} = \text{mass} \times \frac{\text{length}}{(\text{time})^2}$$

∴ The dimension of force is given by

$$F = [M] \times \frac{[L]}{[T]^2} = [MLT^{-2}]$$

1. Distance
   Displacement
   Radius of gyration
   Radius
   Light year
   Parsec
   Astronomical unit

2. Speed
Velocity
Average speed
Average velocity
Terminal velocity
Critical velocity
Velocity of light,
Instantaneous velocity
Escape velocity
Orbital velocity, relative velocity

3. Acceleration

Average acceleration

Instantaneous acceleration

Acceleration due to gravity

Intensity of gravitational field

Centripetal acceleration

Centrifugal acceleration

4. Momentum, impulse

Power

Density

5. Force

Centripetal force

Friction

Gravitational force

Thrust

Viscous drag

Spring force

Magnetic force

Non-conservative force

Radiation force

Tension
Centrifugal acceleration
Normal reaction
Weight
Viscous force
Restoring force
Electrostatic force
Conservative force
Lorentz force

EXAMPLE |1| Dimension of Gravitational Constant

Find out the dimensions of universal gravitational constant used in Newton's law of gravitation.

Sol. According to Newton's law of gravitation, the force F, between two masses m_1 and m_2 separated by distance r can be given as $F = G\frac{m_1 m_2}{r^2}$

Where, G = universal gravitational constant

$$G = \frac{Fr^2}{m_1 m_2} = \frac{\text{Newton} \times (\text{metre})^2}{(\text{kg})^2}$$

$$G = \frac{(\text{mass} \times \text{acceleration}) \times (\text{metre})^2}{(\text{mass})^2}$$

$$= \frac{1}{\text{mass}}\left(\frac{\text{Change in velocity}}{\text{Time}}\right) \times (\text{Length})^2$$

$$G = \frac{(\text{Length})^2}{\text{Mass} \times \text{Time}} \times \frac{\text{Distance}}{\text{Time}}$$

$$G = \frac{[\text{L}]^2}{[\text{M}] \times [\text{T}]} \times \frac{[\text{L}]}{[\text{T}]} = [\text{M}^{-1}\ \text{L}^3\ \text{T}^{-2}]$$

EXAMPLE |2| Pull Over Buddy

Derive the dimensions formula of physical quantities.

(i) Tension

(ii) Velocity gradient

(iii) Linear mass density

(iv) Impulse

Sol. (i) Tension = force = mass × acceleration

$$[M] \times [LT^{-2}] = [MLT^{-2}]$$

(ii) Velocity gradient $= \dfrac{\text{Velocity}}{\text{Distance}} = \dfrac{[LT^{-1}]}{[L]} = [T^{-1}]$

(iii) Linear mass density $= \dfrac{\text{Mass}}{\text{Length}} = \dfrac{[M]}{[L]} = [ML^{-1}]$

(iv) Impulse = force × time $= [MLT^{-2}] \times [T] = [MLT^{-1}]$

DIMENSIONAL ANALYSIS AND ITS APPLICATIONS

1. Checking the dimensional consistency of equations
2. Conversion of one system of units into another
3. Deducing relation among the physical quantities

Checking consistency of equation

The **principle of homogeneity of dimension** states that a physical quantity equation will be dimensionally correct, if the dimensions of all the terms occurring on both sides of the equation are same.

e.g. Let us check the dimensional consistency of the equation of motion as

$$s = ut + \frac{1}{2}at^2$$

$$\left[\frac{1}{2}at^2\right] = [LT^{-2}]/[T^2] = [L]$$

As all the terms on both sides of the equations have the same dimensions, so the given equation is dimensionally correct.

EXAMPLE |3| Test of Consistency

Check whether the given equation is dimensionally correct $\frac{1}{2}mv^2 = mgh$.

[NCERT]

Sol. The dimensions of LHS

$$= [M]\,[LT^{-1}]^2 = [ML^2T^{-2}]$$

The dimensions of RHS $= [M]\,[LT^{-2}][L] = [ML^2T^{-2}]$

The dimensions of LHS and RHS are same and hence the consistency is verified.

EXAMPLE |4| Analysis of an Equation

Check the dimensional consistency of the following equations.

(i) de-Broglie wavelength, $\lambda = \frac{h}{mv}$

(ii) Escape velocity, $v = \sqrt{\frac{2GM}{R}}$

Sol. (i) Given, $\lambda = \frac{h}{mv}$

LHS as wavelength is a distance $\lambda = [L]$

Also RHS, $\frac{h}{mv} = \frac{\text{Planck's constant}}{\text{Mass} \times \text{Velocity}}$

$$= \frac{[ML^2T^{-1}]}{[M] \times [LT^{-1}]} = [L]$$

$\therefore$ LHS = RHS

Hence, the given equation is dimensionally correct.

(ii) Here, $v = \sqrt{\frac{2GM}{R}}$

LHS $v = [LT^{-1}]$ RHS $= \left[\frac{2GM}{R}\right]^{1/2}$

$G = [M^{-1}L^3T^{-2}]$, $R = [L]$, $M = [M]$

$$= \left[\frac{M^{-1}L^3T^{-2}M}{L}\right]^{1/2} = [L^2T^{-2}]^{1/2} = [LT^{-1}]$$

$\therefore$ Dimensions of LHS = Dimensions of RHS

Hence, the equation is dimensionally correct.

Q NO. The equation $\left(P + \frac{a}{V^2}\right)(V - b)$ constant. The units of a are **(2008)**

A $Dyne \times \mathrm{cm}^5$

B $Dyne \times \mathrm{cm}^4$

C $Dyne/\mathrm{cm}^3$

D $Dyne/\mathrm{cm}^2$

Q NO. The velocity v of a particle at time t is given by $v = at + \frac{b}{t+c}$, where a, b and c are constants. The units of a, b and c are (2006)

A m, ms and ms^{-2}

B ms^{-2}, m and s

C m^2, s and ms^{-2}

D ms^{-2}, ms and m

Hint :-
focus on (+, −, =)

If A, B, C are three different physical Quantities, Which of the following are meaningful and meaningless

we Can

1) $A + B$

2) $A - C$

3) $A \times B$

4) $A \times B \times C$

5) $(A \times B)/C$

6) $A + BC$

Conversation of. Units

i.e. $$n_1 u_1 = n_2 u_2 \quad \Rightarrow \quad n_2 = \frac{n_1 u_1}{u_2} \qquad \ldots\text{(i)}$$

Where, u_1 and u_2 are two units of measurement of the quantity and n_1 and n_2 are their respective numerical values. If M_1, L_1 and T_1 are the fundamental units of mass, length and time in one system and while for other system, M_2, L_2 and T_2 are the fundamental units of mass, length and time then $u_1 = [M_1^a L_1^b T_1^c]$ and $u_2 = [M_2^a L_2^b T_2^c]$

From Eq. (i) $$n_2 = \frac{n_1 [M_1^a L_1^b T_1^c]}{[M_2^a L_2^b T_2^c]} = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c$$

$$\boxed{n_2 = n_1 \left[\frac{M_1}{M_2}\right]^a \left[\frac{L_1}{L_2}\right]^b \left[\frac{T_1}{T_2}\right]^c}$$

EXAMPLE |5| Energy Estimation

A calorie is a unit of heat or energy and it equals about 4.2 J, where $1\text{ J} = 1\text{ kg-m}^2/\text{s}^2$. Suppose we employ a system of units in which the unit of mass equals α kg, the unit of length is β m, the unit of time is γ s. Show that a calorie has a magnitude $4.2\,\alpha^{-1}\beta^{-2}\gamma^2$ in terms of new units.

[NCERT]

Sol. The dimensional formula of energy $= [ML^2T^{-2}]$

Let M_1, L_1, T_1 and M_2, L_2, T_2 are the units of mass, length and time in given two systems.

$$\therefore \quad M_1 = 1\text{ kg}, M_2 = \alpha\text{ kg}$$
$$L_1 = 1\text{ m}, L_2 = \beta\text{ m}$$
$$T_1 = 1\text{ s}, T_2 = \gamma\text{ s}$$

For any physical quantity, the product of its magnitude and unit is always constant.

$$n_1u_1 = n_2u_2$$

$$\text{or} \quad n_2 = n_1\frac{u_1}{u_2} = 4.2 \times \frac{[M_1L_1^2T_1^{-2}]}{[M_2L_2^2T_2^{-2}]}$$

$$= 4.2\left[\frac{M_1}{M_2}\right] \times \left[\frac{L_1}{L_2}\right]^2 \times \left[\frac{T_1}{T_2}\right]^{-2}$$

$$= 4.2\left[\frac{1}{\alpha}\text{kg}\right] \times \left[\frac{1}{\beta}\text{m}\right]^2 \times \left[\frac{1}{\gamma}\text{s}\right]^{-2}$$

$$n_2 = 4.2\alpha^{-1}\beta^{-2}\gamma^2 \text{ new unit}$$

$$\therefore \quad 1\text{ cal} = 4.2\alpha^{-1}\beta^{-2}\gamma^2 \text{ new unit}$$

CONVERSION OF UNITS

EXAMPLE |6| Power Estimation

Find the value of 60J per min on a system that has 100 g, 100 cm and 1 min as the base units.

Sol. Given, $P = \frac{60 \text{ joule}}{1 \text{min}} = \frac{60 \text{ joule}}{60 \text{ s}} = 1 \text{ watt}$

which is the SI unit of power

Dimensional formula of power is $[ML^2T^{-3}]$.

$\therefore \quad a = 1, b = 2$ and $c = -3$

SI	New System
$n_1 = 1$	$n_2 = ?$
$M_1 = 1 \text{ kg} = 1000 \text{ g}$	$M_2 = 100 \text{ g}$
$L_1 = 1 \text{ m} = 100 \text{ cm}$	$L_2 = 100 \text{ cm}$
$T_1 = 1 \text{ s}$	$T_2 = 1 \text{ min} = 60 \text{ s}$

.2

of

3. Deducing Relation among the Physical Quantities

Suppose, the time period t of oscillation of the simple pendulum depends on its length (l), mass of the bob (m) and acceleration due to gravity (g).

Let $$t = k m^a l^b g^c \quad \ldots \text{(i)}$$

Where a, b, c are the dimensions and k is dimensionless constant of proportionality.

Considering dimensions on both sides in terms of M, L, T, we get

$$[M^0L^0T^1] = M^aL^b[LT^{-2}]^c$$
$$= M^aL^{b+c}T^{-2c}$$

Applying the principle of homogeneity of dimensions, we get

$$a = 0, \; -2c = 1 \quad \Rightarrow \quad c = \frac{-1}{2},$$

$$b + c = 0 \quad \Rightarrow \quad b = -c \quad \Rightarrow b = \frac{1}{2}$$

Substituting the values of a, b and c in Eq. (i), we get

0 g,

$$t = k m^0 l^{1/2} g^{-1/2} = k\sqrt{\frac{l}{g}}$$

$$\Rightarrow \quad t = k\sqrt{\frac{l}{g}}$$

Limitations of Dimensional Analysis

(i) It does not give any information whether a physical quantity is a scalar or a vector.

(ii) It gives no information about the dimensionless constant in the formula e.g. 1, 2, 3 … π etc.

(iii) We cannot drive the formula containing the trigonometrical function logarithmic function, exponential function which have no dimensions.

(iv) If a quantity depends on more than three factors, having dimensions, the formula cannot be derived.

This is because, equating the powers of M, L and T on either side of the dimensional equation, then we can obtain three equations from which we can compute three unknown dimensions.

EXAMPLE |8| A Stretched Spring

A body of mass m hung at one end of the spring executes SHM. Prove that the relation $T = 2\pi m/k$ is incorrect, where k is the force constant of the spring. Also, derive the correct relation.

Sol. It is given that $T = \frac{2\pi m}{k}$

LHS, $T = [T]$

RHS, $\frac{2\pi m}{k} = \frac{[M]}{[MT^{-2}]} = [T^2]$

$$\left[\because k = \frac{\text{Force}}{\text{Length}} = \frac{[MLT^{-2}]}{[L]} = [MT^{-2}]\right]$$

$\therefore$ LHS $\neq$ RHS

Hence, the relation is incorrect.

To find the correct relation, suppose $T = km^a k^b$, then

$$[T]^1 = [M]^a [MT^{-2}]^b = M^{a+b}T^{-2b}$$

$\therefore$ $a + b = 0, -2b = 1$

On solving, we get $b = \frac{-1}{2}, a = \frac{1}{2}$

$\therefore$ $T = km^{1/2}k^{-1/2}$

Hence, $T = k\sqrt{\frac{m}{k}}$

The End