Properties of Conductors

(i) Inside a conductor, electrostatic field is zero.
(ii) At the surface of a charged conductor, electrostatic field must be normal to the surface at every point.
(iii) The interior of the conductor can have no excesscharge in the static situation.
(iv) Electrostatic potential is constant throughout the volume of the conductor and has the same value(as inside) on its surface.
(v) Surface charge density of a conductor could be different at different points.

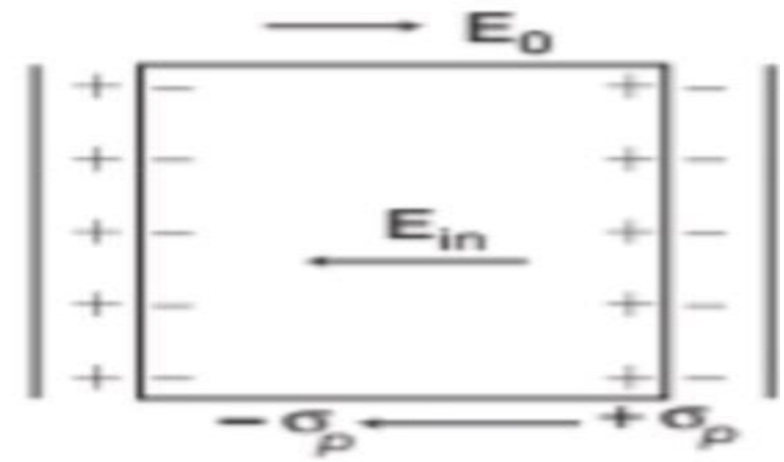
Electrostatic Shielding

The phenomenon of protecting a certain region of space from external electric field is called electrostatic shielding. We know that inside a conductor, electric field is zero, so to protect some instruments from external field, they are enclosed in hollow conductors.

Dielectrics and Polarisation

Dielectrics (or insulators) are non-conducting substances. In contrast to conductors, they have no (or negligible number of) free charges or charge carriers.

In a dielectric under the effect of an external field, a net dipole moment is induced in the dielectric. Due to molecular dipole moments, a net charge appears on the surface of the dielectric.



Polarised dielectric slab

These induced charges (of densities $-\sigma_p$ and and $+\sigma_p$) produce a field opposing the external field. Induced field is lesser in magnitude than the external field. So, field inside the dielectric gets reduced.

$$E = |E_0| - |E_{in}|$$

where, E = resultant electric field in the dielectric,

E_0 =external electric field between two plates

and E_{in} = electric field inside the dielectric.

A net dipole moment is developed by an external field in either case, whether a polar or non-polar dielectric.

Dielectric Constant (K)

The ratio of the strength of the applied electric field to the strength of the reduced value of the electric field on placing the dielectric between the two plates is called the dielectric constant of the dielectric medium.

It is also known as **relative permittivity** or **specific inductive capacity** and is denoted by K (or ε_r).

Therefore, dielectric constant of a dielectric medium is given by

$$\boxed{K = \frac{E_0}{E}}$$

Note The value of K is always greater than 1.

Polarisation (P)

The induced dipole moment developed per unit volume in a dielectric slab on placing it in an electric field is called polarisation. It is denoted by P. If p is induced dipole moment acquired by an atom of the dielectric and N is the number of atoms per unit volume, then polarisation is given by

$$\boxed{P = Np}$$

The induced dipole moment (p) acquired by the atom is found to be directly proportional to the reduced value of electric field (E) and is given by

$$\boxed{p = \alpha\varepsilon_0 \mathbf{E}}$$

where, α is constant of proportionality and is called **atomic polarisability.**

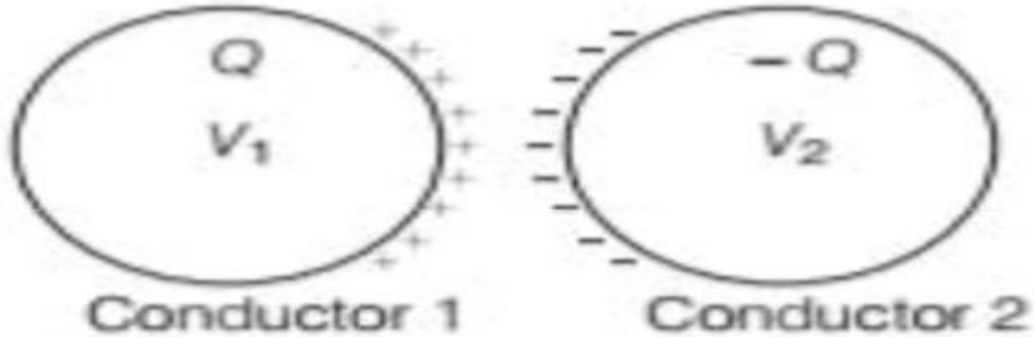
Electric Susceptibility (χ)

The polarisation density of a dielectric slab is directly proportional to the reduced value of the electric field and may be expressed as $\boxed{P = \chi\varepsilon_0 E}$

The constant of proportionality χ is called **electric susceptibility** of the dielectric slab. It is a dimensionless constant.It describes the electrical behaviour of a dielectric.

Capacitors and Capacitance

A capacitor is a system of two conductors separated by an insulating medium. The conductors have charges Q and $-Q$ with potential difference, $V = V_1 - V_2$ between them. The electric field in the region between the conductors is proportional to the charge Q.



A system of two conductors or capacitors

If the potential difference (V) is the work done per unit positive charge in taking a small test charge from the conductor 2 to 1 against the field, then V is proportional to Q and the ratio $\frac{Q}{V}$ is a constant.

$$\boxed{C = \frac{Q}{V}}$$

The constant C is called the capacitance of the capacitor. Capacitance C depends on shape, size and separation of the system of two conductors. The SI unit of capacitance is farad. Its dimensional formula is $[M^{-1}L^{-2}T^{4}A^{2}]$.

$$1 \text{ farad} = 1 \text{ coulomb/volt}$$

A capacitor with fixed capacitance is symbolically shown as , while the one with variable capacitance is shown as . In practice, farad is a very big unit, the most common units are its sub-multiples.

$$1 \mu F = 10^{-6} \text{ F}, 1 \text{ nF} = 10^{-9} \text{ F}, 1 \text{ pF} = 10^{-12} \text{ F}$$

EXAMPLE |1| When 1×10^{12} electrons are transferred from one conductor to another, a potential difference of 10 V appears between the conductors. Find the capacitance of the two conductors.

Sol. Given, number of electrons,

$$n = 1 \times 10^{12}$$

$\therefore$ Charge transferred,

$$Q = ne = 1 \times 10^{12} \times 1.6 \times 10^{-19}$$

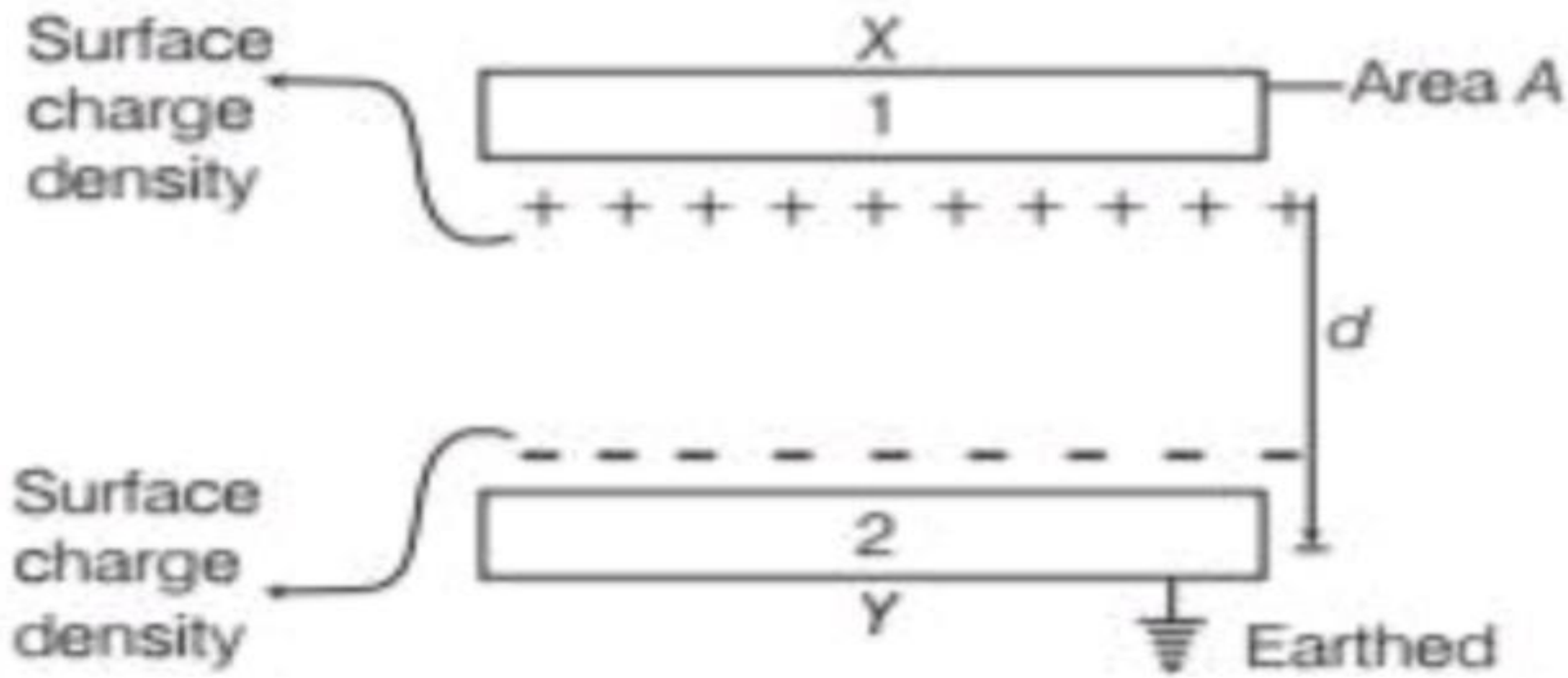
$$= 1.6 \times 10^{-7} \text{ C} \qquad [\because e = 1.6 \times 10^{-19} \text{ C}]$$

$\therefore$ Capacitance between two conductors,

$$C = \frac{Q}{V} = \frac{1.6 \times 10^{-7}}{10}$$

$$= 1.6 \times 10^{-8} \text{ F}$$

PARALLEL PLATE CAPACITOR



Suppose the plate X is given a charge of $+q$ coulomb. By induction, $-q$ coulomb of charge is produced on the inner surface of the plate Y and $+q$ coulomb on the outer surface. Since, the plate Y is connected to the earth, the $+q$ charge on the outer surface flows to the earth. Thus, the plates X and Y have equal and opposite charges.

The intensity of electric field between the plates will be given by

$$E = \frac{\sigma}{\varepsilon_0}$$

Areal charge density

The charge on each plate is q and the area of each plate is A. Thus,

$$\sigma = \frac{q}{A} \text{ and so, } E = \frac{q}{\varepsilon_0 A} \qquad \text{...(i)}$$

Now, let the potential difference between the two plates be V volt. Then, the electric field between the plates is given by

$$E = \frac{V}{d} \text{ or } V = Ed$$

Substituting the value of E from Eq. (i), we get

$$V = \frac{qd}{\varepsilon_0 A}$$

$\therefore$ Capacitance of the parallel plate capacitor is given by

$$C = \frac{q}{V} = \frac{q}{qd/\varepsilon_0 A} \quad \text{or} \quad \boxed{C = \frac{\varepsilon_0 A}{d}}$$

where, $\varepsilon_0 = 8.85 \times 10^{-12}$ F/m

It is clear from this formula that in order to obtain high capacitance.

(i) A should be large, i.e. the plates of large area should be taken.

(ii) d should be small, i.e. the plates should be kept close to each other.

Note Capacity of an isolated spherical conductor is

$$\boxed{C = 4\pi\varepsilon_0 r}$$

where, $r =$ radius of the sphere.

EXAMPLE |2| What is the area of the plates of a 2F parallel plate capacitor, given that the separation between the plates is 0.5 cm? (You will realise from your answer why ordinary capacitors are in the range of μF or less. However, electrolytic capacitors do have a much larger capacitance (0.1 F) because of very minute separation between the conductors). **NCERT**

Sol. Given, capacitance, $C = 2\ \text{F}$

and separation between plates, $d = 0.5\ \text{cm} = 0.5 \times 10^{-2}\ \text{m}$

Capacitance of a parallel plate capacitor,

$$C = \frac{\varepsilon_0 A}{d}$$

$$\text{or } A = \frac{Cd}{\varepsilon_0} = \frac{2 \times 0.5 \times 10^{-2}}{8.854 \times 10^{-12}}$$

$$= 1.13 \times 10^{9}\ \text{m}^2$$

$$= 1130\ \text{km}^2$$

This area is very large, so it is not possible that the capacitance of a capacitor is too large as 2 F.

So, the capacitance of any capacitor should be the range of 2 μF.

EXAMPLE |3| A parallel plate capacitor has plate area $25\ \text{cm}^2$ and a separation of 2 mm between the plates. The capacitor is connected to a battery of 12 V.

(i) Find the charge on the capacitor.

(ii) If the plate separation is decreased to 1.0 mm, then find the extra charge given by the battery to the positive plate.

Sol. Given, area of plate, $A = 25\ \text{cm}^2 = 25 \times 10^{-4}\ \text{m}^2$

Distance between the plates, $d = 2\ \text{mm} = 2 \times 10^{-3}\ \text{m}$

Potential difference, $V = 12\ \text{V}$

(i) Charge on the capacitor, $q = CV$

$$= \frac{\varepsilon_0 A}{d} V = \frac{8.85 \times 10^{-12} \times 25 \times 10^{-4} \times 12}{2 \times 10^{-3}}$$

$$= 1.33 \times 10^{-10}\ \text{C}$$

(ii) If the plate separation is decreased to half, the capacity becomes twice. Then, charge becomes twice as battery is still connected.

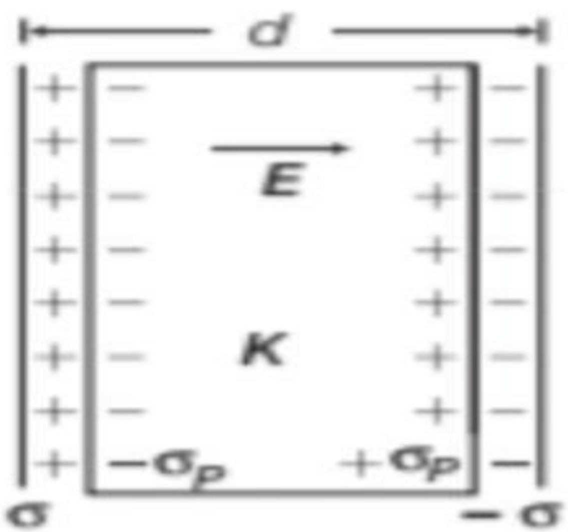
$\therefore$ Extra charge given by the battery $= q' - q$

$$= 2q - q = q = 1.33 \times 10^{-10}\ \text{C}$$

Effect of Dielectric on Parallel Plate Capacitor

Consider a dielectric is inserted between the plates of a parallel plate capacitor and fully occupying the intervening region as shown in figure. The dielectric is polarised by the field, with surface charge densities σ_P. and $-\sigma_P$.

The electric field in the dielectric then corresponds to the case when the net surface charge density on the plates is $\pm(\sigma - \sigma_P)$.



Dielectric between the plates of a capacitor

So, net electric field between the plates, $E = \dfrac{\sigma - \sigma_P}{\varepsilon_0}$

[$\because$ dielectric is polarised in the opposite direction of external field]

$\therefore$ Potential difference between the plates,

$$V = Ed = \frac{\sigma - \sigma_P}{\varepsilon_0} d$$

For linear dielectrics, we expect σ_p to be proportional to E_0 i.e. to σ.

Thus, $(\sigma - \sigma_p)$ is proportional to σ and we can write,

$$\sigma - \sigma_p = \frac{\sigma}{K}$$

where, K is a constant characteristics of the dielectric.

Clearly, $K > 1$ $[\because \sigma_p < \sigma]$

then, $$V = \frac{\sigma d}{\varepsilon_0 K} = \frac{qd}{A\varepsilon_0 K}$$

$\therefore$ The capacitance C with dielectric between the plates is given by

$$\boxed{C = \frac{q}{V} = \frac{\varepsilon_0 KA}{d}}$$

The product $\varepsilon_0 K$ is called the **permittivity of the medium** and is denoted by ε.

$$\varepsilon = \varepsilon_0 K$$

For vacuum, $K = 1$ and $\varepsilon = \varepsilon_0$, where ε_0 is called the **permittivity of the vacuum.**

The dimensionless ratio,

$$K = \frac{\varepsilon}{\varepsilon_0}$$

is called the **dielectric constant** of the substance.

Similarly, $$K = \frac{C}{C_0}$$

Thus, the dielectric constant of a substance is the factor ($K > 1$) by which the capacitance increases from its vacuum value, when the dielectric is inserted fully between the plates of a capacitor.

(i) When a dielectric slab of thickness t is inserted between the plates, then

Capacitance, $$C = \frac{\varepsilon_0 A}{d - t + \frac{t}{K}}$$

(ii) If several slabs of dielectric constants $K_1, K_2, K_3, \ldots$ and respective thicknesses $t_1, t_2, t_3, \ldots$ are placed in between the plates of a capacitor, then capacitance,

$$C = \frac{\varepsilon_0 A}{d - (t_1 + t_2 + t_3 + \ldots) + \frac{t_1}{K_1} + \frac{t_2}{K_2} + \frac{t_3}{K_3} + \ldots}$$

(iii) If a metallic slab $(K = \infty)$ of thickness t is placed between the plates of capacitor, then

Capacitance,

lic slab ($K = \infty$) of thickness t is placed
he plates of capacitor, then

ce,

$$C = \frac{\varepsilon_0 A}{d - t}.$$

slab fills the entire space between the
. $d = t$), then capacitance will become

EXAMPLE |4| In a parallel plate capacitor with air between the plates, each plate has an area of $6 \times 10^{-3} \, m^2$ and the separation between the plates is 3 mm.

(i) Calculate the capacitance of the capacitor.

(ii) If this capacitor is connected to 100 V supply, what would be the charge on each plate?

(iii) How would charge on the plates be affected if a 3 mm thick mica sheet of $K = 6$ is inserted between the plates while the voltage supply remains connected?

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Sol. Given, area of each plate, $A = 6 \times 10^{-3}\,\text{m}^2$

Distance between the plates,

$$d = 3\text{ mm}$$
$$= 3 \times 10^{-3}\,\text{m}$$

(i) Capacitance of parallel plate capacitor is given by

$$C = \frac{\varepsilon_0 A}{d} = \frac{8.85 \times 10^{-12} \times 6 \times 10^{-3}}{3 \times 10^{-3}}$$

$\therefore$ $$C = 1.77 \times 10^{-11}\,\text{F}$$

(ii) Charge on parallel plate capacitor is given by

$$Q = CV = 1.77 \times 10^{-11} \times 100$$
$$= 1.77 \times 10^{-9}\,\text{C}$$

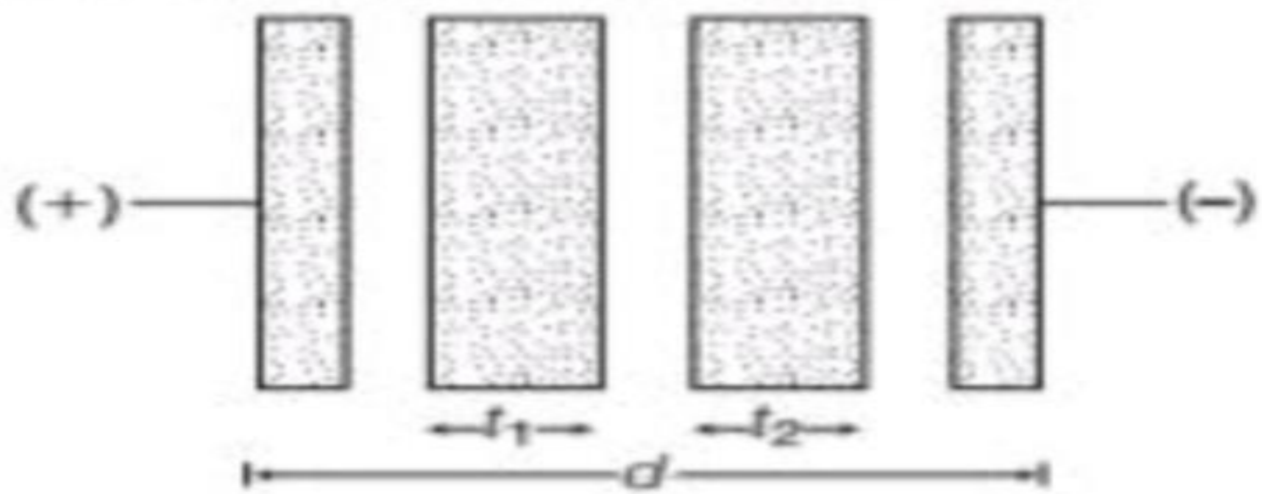
(iii) Given, $K = 6$

Now, $C' = KC$

$\Rightarrow$ $$\frac{Q'}{V} = \frac{KQ}{V}$$

$\therefore$ $$Q' = KQ$$
$$= 6 \times 1.77 \times 10^{-9}$$
$$= 10.62 \times 10^{-9}\,\text{C}$$

EXAMPLE |5| An air-cored capacitor of plate area A and separation d has a capacity C. Two dielectric slabs are inserted between its plates in two different manners as shown. Calculate the capacitance in it.



Sol. Let the charges on the plates are Q and $-Q$.

Electric field in free space is $E_0 = \dfrac{\sigma}{\varepsilon_0} = \dfrac{Q}{A\varepsilon_0}$.

Electric field in first slab is $E_1 = \dfrac{E_0}{K_1} = \dfrac{Q}{A\varepsilon_0 K_1}$.

Electric field in second slab is $E_2 = \dfrac{E_0}{K_2} = \dfrac{Q}{A\varepsilon_0 K_2}$.

The potential difference between the plates is

$$V = E_0(d - t_1 - t_2) + E_1 t_1 + E_2 t_2$$