CHAPTER 2
ELECTROSTATIC POTENTIAL AND CAPACITANCE

CHAPTER CHECKLIST

1. Electrostatic Potential,
2. Electrostatic PotentialDifference and
3. Electrostatic Potential Energy
4. Dielectric and Capacitance

Electrostatic potential

The electrostatic potential at any point in the region of electric field is equal to the amount of work done in bringing a unit positive test charge (without acceleration) from infinity to that point.

Electrostatic potential (V) =Work done (W)/ Charge(qo)

Work done ([W]) by an external force in bringing (without acceleration) a unit positive charge from infinity to a point is equal to the potential (V) at thatpoint,

i.e. $$V = \frac{[W_\infty]_{ext}}{q_0} = \frac{-[W_\infty]_{elec}}{q_0} \qquad [\because [W_\infty]_{ext} = -[W_\infty]_{elec}]$$

(a)It is a scalar quantity. Its SI unit is volt (V) and 1V = 1 J/C and its dimensional formula is $ML^2T^{-3}A^{-1}$

Note Electric potential is state dependent function as electrostaticforces are conservative forces. No work is done in moving a unitpositive test charge over a closed path in an electric field

EXAMPLE |1| Potential at a point P in space is given as 3×10^5 V. Find the work done in bringing a charge of 2×10^{-6} C from infinity to the point P. Does the answer depend on the path along which the charge is brought?

Sol. Given,

Potential at the point P,

$$V = 3 \times 10^5 \text{ V, charge, } q_0 = 2 \times 10^{-6} \text{ C}$$

Work done in bringing the charge from infinity to the point P is

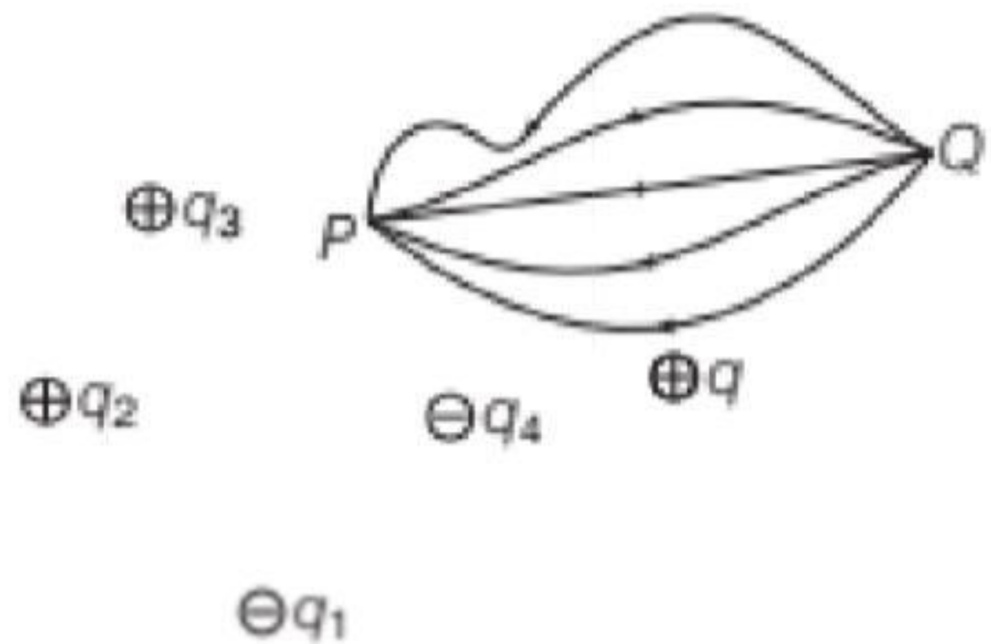
$$W_\infty = q_0 V = 2 \times 10^{-6} \times 3 \times 10^5$$

$$= 6 \times 10^{-1} = 0.6 \text{ J}$$

No, the work done will be path independent.

ELECTROSTATIC POTENTIAL DIFFERENCE

- Electrostatic potential difference between two points P and Q of a charge configuration consisting of charges q1,q2,q3,q4 and q is equal to the work done by an external force in moving a unit positive test charge against the electrostatic force from point Q to P along any path between these two points



Electrostatic potential difference between two points *P* and *Q*

Electrostatic potential difference between two points *P* and *Q*

If V_Q and V_P are the electrostatic potentials at *Q* and *P* respectively, then electrostatic potential difference between points *Q* and *P* is

$$\Delta V = V_P - V_Q$$

Thus,

$$\boxed{\Delta V = \frac{W_{QP}}{q_0}}$$

$$\Delta V = \frac{W_{QP}}{q_0} = \frac{[\mathrm{ML^2T^{-2}}]}{[\mathrm{AT}]} = [\mathrm{ML^2T^{-3}A^{-1}}]$$

The SI unit of electrostatic potential difference is **volt.**

$$1\,\mathrm{V} = 1\,\mathrm{JC^{-1}} = 1\,\mathrm{N\text{-}m\,C^{-1}}$$

Note One electron-volt (1 eV) is the energy equal to the work required to move a single elementary charge e such as an electron or the proton through a potential difference of exactly one volt (1 V).

$$\therefore \quad 1\,\text{eV} = e\,(1\text{V}) = (1.60 \times 10^{-19}\,\text{C})\,(1\,\text{J/C}) = 1.60 \times 10^{-19}\,\text{J}$$

EXAMPLE |2| The potential difference between two points is 20 V. How much work will be done in carrying a charge of 400 μC from one point to the another?

Sol. Given, $\Delta V = 20\,\text{V}$ and $q = 400\,\mu\text{C} = 400 \times 10^{-6}\,\text{C}$

We know that,

$$\text{Electrostatic potential difference} = \frac{\text{Work done}}{\text{Charge}}$$

$$\Rightarrow \quad \Delta V = \frac{W}{q}$$

$$\Rightarrow \quad 20 = \frac{W}{400 \times 10^{-6}}$$

$$\therefore \quad W = 20 \times 400 \times 10^{-6} = 8 \times 10^{-3}\,\text{J}$$

EXAMPLE |3| If 100 J of work must be done to move an electric charge of magnitude 4 C from a place *A*, where potential is –10 V to another place *B* where potential is *V* volt. Find the value of *V*.

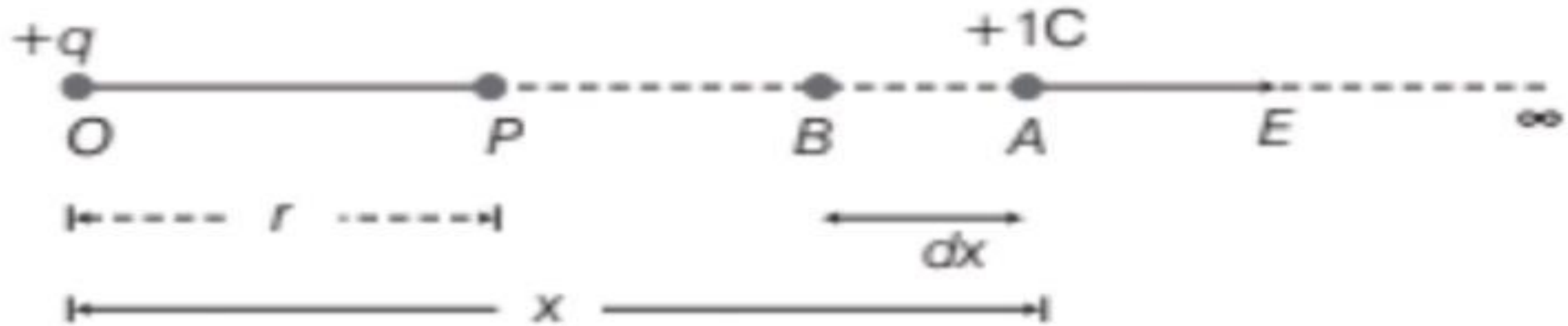
Sol. Given, $W_{AB} = 100\text{ J},\ q = 4\text{ C},\ V_A = -10\text{V},\ V_B = V = ?$

Since, $W_{AB} = q(V_B - V_A)$

$\Rightarrow \quad 100 = 4(V + 10) \Rightarrow V = 15\text{ V}$

ELECTROSTATIC POTENTIAL DUE TO A POINT CHARGE

Let P be the point at a distance r from the origin O at which the electric potential due to charge $+q$ is required.



Let A be an intermediate point on this path where OA = x. The electrostatic force on a unit positive charge at A is given by

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q \times 1}{x^2} \qquad \text{[along } OA\text{]} \quad \ldots \text{(i)}$$

Small work done in moving the charge through a distance dx from A to B is given by

dw =F.dx= Fdx cos 180°=-Fdx [cos 180°=-1]

⇒dW=- Fdx ...(ii) T

⇒otal work done in moving a unit positive charge from ∞ to the point P is given by

$$W = \int_{\infty}^{r} -F\,dx$$

$$= \int_{\infty}^{r} -\frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{x^2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\left[\frac{-1}{x}\right]_{\infty}^{r} \qquad \left[\because \int x^{-2}\,dx = -\frac{1}{x}\right]$$

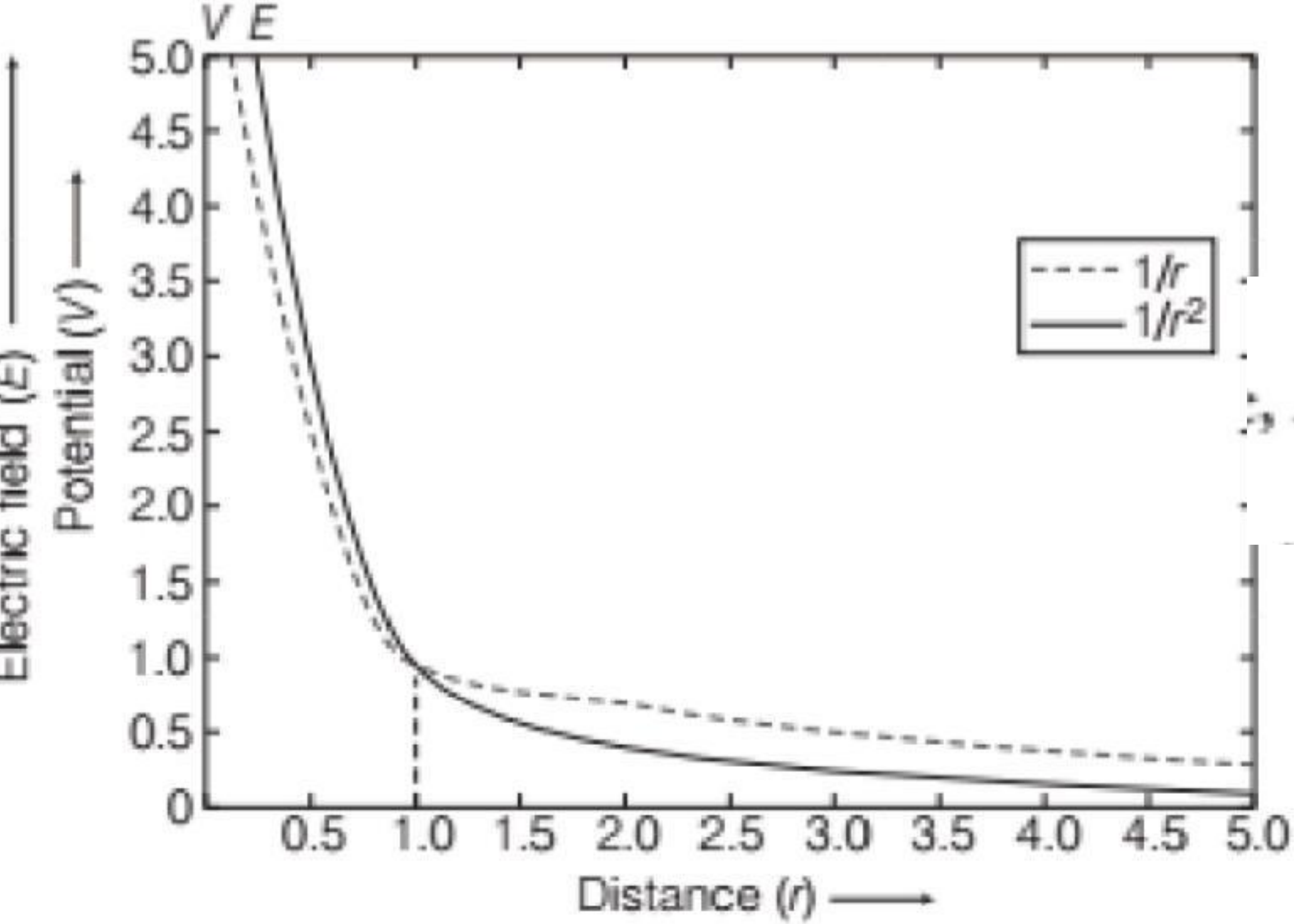
$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r} - \frac{1}{\infty}\right]$$

$$W = \frac{q}{4\pi\varepsilon_0 r} \qquad \ldots(iii)$$

From the definition of electric potential, this work is equal to the potential at point P.

$$V = \frac{q}{4\pi\varepsilon_0 r} \qquad \text{...(iv)}$$

A positively charged particle produces a positive electric potential. A negatively charged particle produces a negative electric potential. Here, we assume that electrostatic potential is zero at infinity



, $F \propto \frac{1}{r^2}$, $E \propto \frac{1}{r^2}$ but $V \propto \frac{1}{r}$,

tion of electrostatic potential V and electric field E with distance r

- **EXAMPLE [4] What is the electrostatic potential at the surface of a silver nucleus of diameter 12.4 fermi? Atomicnumber (Z) for silver is 47.**

Sol : Given, r =6.2 fermi = 6.2×10^{-15} m and Z = 472..

Charge of the nucleus, q= Ze = 47 x1.6x10-19 C [:e=1.6×10-19 C]..

Electrostatic potential at the surface,

Sol. Given, $r = \frac{12.4}{2} = 6.2$ fermi $= 6.2 \times 10^{-15}$ m and $Z = 47$

$\therefore$ Charge of the nucleus, $q = Ze = 47 \times 1.6 \times 10^{-19}$ C

$[\because e = 1.6 \times 10^{-19}\text{ C}]$

$\therefore$ Electrostatic potential at the surface,

$$V = \frac{q}{4\pi\varepsilon_0 r} = \frac{9 \times 10^9 \times 47 \times 1.6 \times 10^{-19}}{6.2 \times 10^{-15}} = 1.09 \times 10^7 \text{V}$$

EXAMPLE |5| A point charge Q is placed at point O as shown in the figure. Is the potential difference $(V_A - V_B)$ positive, negative or zero, if Q is

(i) positive? (ii) negative?

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Sol. Let the distance of points A and B from charge Q l
are r_A and r_B, respectively.

$\therefore$ Potential difference between points A and B,

$$V_A - V_B = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$$

As, $\quad r_A = OA, \; r_B = OB$ and $r_A < r_B$

$$\Rightarrow \quad \frac{1}{r_A} > \frac{1}{r_B}$$

Therefore, $\left[\frac{1}{r_A} - \frac{1}{r_B}\right]$ has positive value.

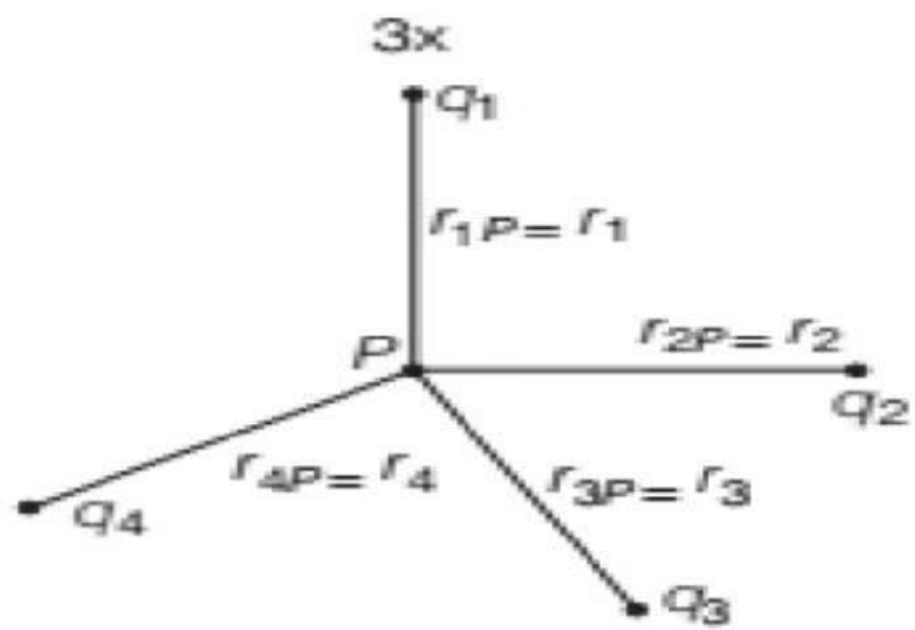
$(V_A - V_B)$ depends on the nature of charge Q.

(i) $(V_A - V_B)$ is positive when $Q > 0$, then

(ii) $(V_A - V_B)$ is negative when $Q < 0$.

ELECTROSTATIC POTENTIAL DUE TO A SYSTEM OF CHARGES

Let there be a number of point charges $q_1, q_2, q_3, \ldots, q_n$ at distances $r_1, r_2, r_3, \ldots, r_n$ respectively from the point P, where electric potential is to be calculated.



A system of charges

Similarly, $V_2 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_{2P}}$, $V_3 = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_3}{r_{3P}}$, ...,

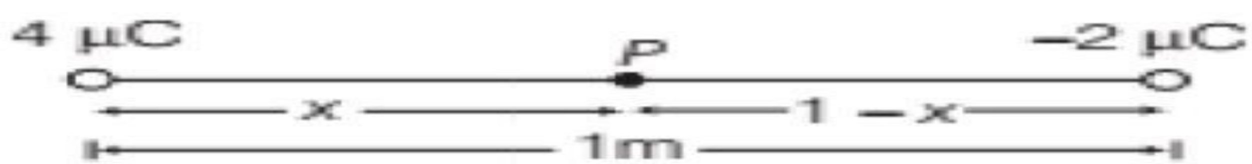
$$V_n = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_n}{r_{nP}}$$

Using superposition principle, we obtain resultant potential at point P due to total charge configuration as the algebraic sum of the potentials due to individual charges.

$\therefore \quad V = V_1 + V_2 + V_3 + \cdots + V_n$

EXAMPLE [6] Two point charges of 4 μC and -2 μC areseparated by a distance of 1 m in air. Find the location of a point on the line joining the two charges, where the electric potential is zero.

Sol Let the electrostatic potential be zero at point P between the two charges separated by a distance x metre.



At point P, $$V_P = V_1 + V_2 = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{r_2} = 0$$

$$\Rightarrow \quad \frac{1}{4\pi\varepsilon_0}\cdot\frac{4\times10^{-6}}{x} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{(-2\times10^{-6})}{(1-x)} = 0$$

$$\Rightarrow \quad \frac{4\times10^{-6}}{x} = \frac{2\times10^{-6}}{(1-x)}$$

$$\Rightarrow \quad \frac{4}{x} = \frac{2}{(1-x)}$$

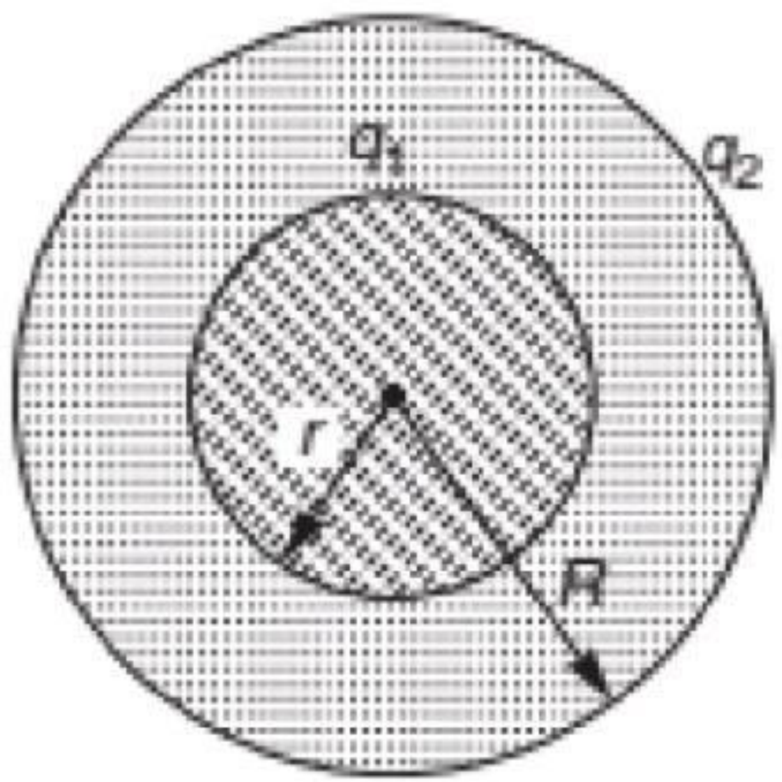
$$\Rightarrow \quad 2(1-x) = x$$

$$\Rightarrow \quad 2 = 3x \quad \text{or} \quad x = \frac{2}{3}$$

∴ Electrostatic potential is zero at a distance 2/3 m from charge 4μC between the two charges.

- EXAMPLE |7| A charge Q is distributed over two concentric hollow spheres of radii r and R (>r) such that the surface densities are equal. Find the potential at the common centre.

Sol Let q, andq, be the charges on them.



$$\sigma_1 = \sigma_2$$

$$\therefore \quad \frac{q_1}{4\pi r^2} = \frac{q_2}{4\pi R^2}$$

$$\therefore \quad \frac{q_1}{q_2} = \frac{r^2}{R^2}$$

i.e. charge on them is distributed in above ratio

or $q_1 = \frac{r^2}{r^2 + R^2} Q$ and $q_2 = \frac{R^2}{r^2 + R^2} Q$

∴ Potential at centre

$$V = \text{Potential due to } q_1 + \text{Potential due to } q_2$$

$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_1}{r} + \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_2}{R} = \frac{Q(R+r)}{4\pi\varepsilon_0(r^2+R^2)}$$

EXAMPLE [8] Two spherical metal shells with different radii r and R are far apart and connected by a thin conducting wire. A charge Q is placed on one of them. The charge redistributes so that same is on each sphere. How much charge is on the sphere with radius r?

Sol **The electrical potential of a spherical shell with charge q and radius r is kq/r, where k = 1/(4π ε)**

Since, the shells are joined by a conductor the charge will distribute between them so that they attain the same electrical potential.

Let the charge on the sphere with radius r be q, and that on the another sphere q_R. Then, equating the potentials gives $q_r/r=q_R/R$

$\Rightarrow$ $$q_r = q_R (r/R) \quad ...(i)$$

$\because$ The total charge equals the original charge.

$\therefore$ $$Q = q_r + q_R \Rightarrow q_R = Q - q_r$$

By Eq. (i), $$q_r = (Q - q_r)(r/R)$$

Solving for q_r gives $q_r(1 + r/R) = Q(r/R)$

$\Rightarrow$ $$q_r = Qr/(R + r)$$

which is the required charge.

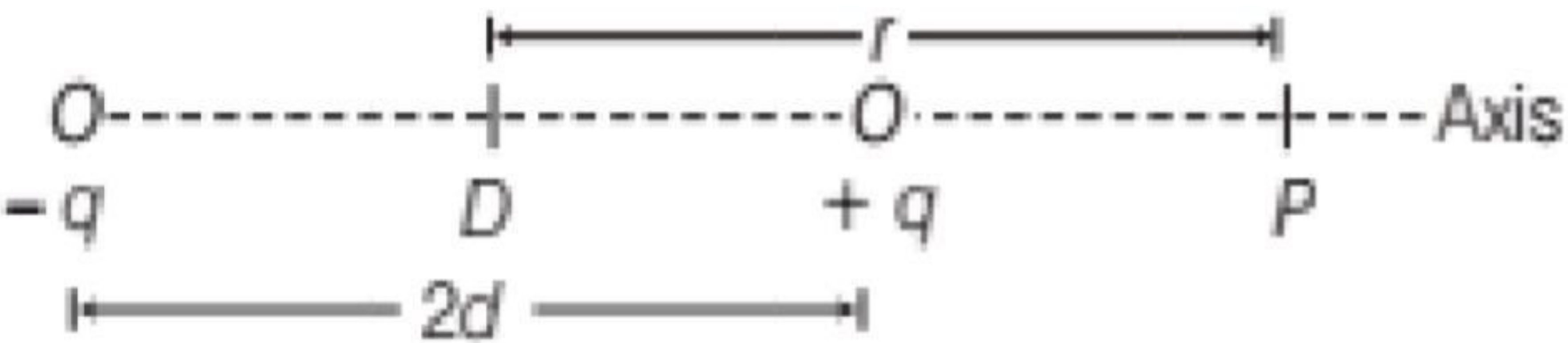
Note The value of k is not needed, stating proportionally is sufficient.

Electrostatic Potential due to Dipole on its Axis and Equatorial Plane

Detail of proof is given in class notes

On the dipole axis, $\theta = 0° \text{ or } \pi$

$$\therefore \quad V = \pm \frac{p}{4\pi\varepsilon_0 r^2}$$

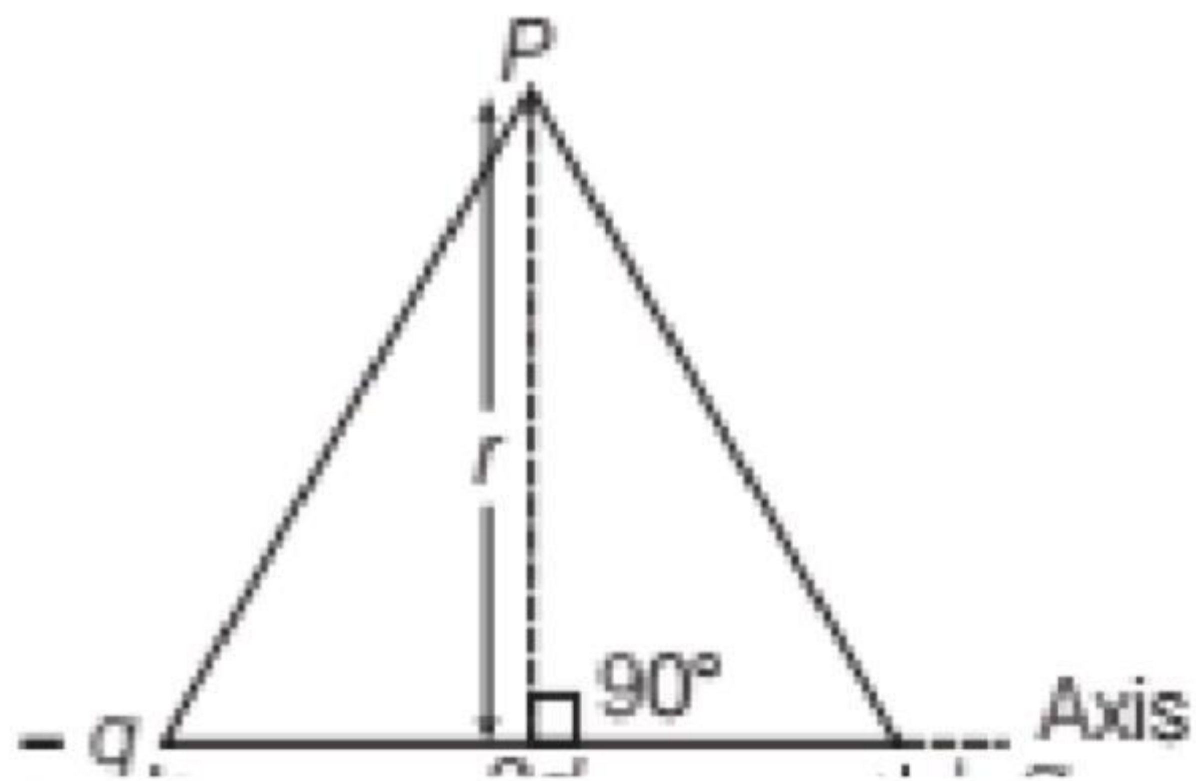


Positive sign for $\theta = 0°$ and negative sign for $\theta = \pi$.

In the equatorial plane, $\theta = \frac{\pi}{2}$

$$\cos\theta = \cos\frac{\pi}{2} = 0$$

$$\therefore \quad V = 0$$



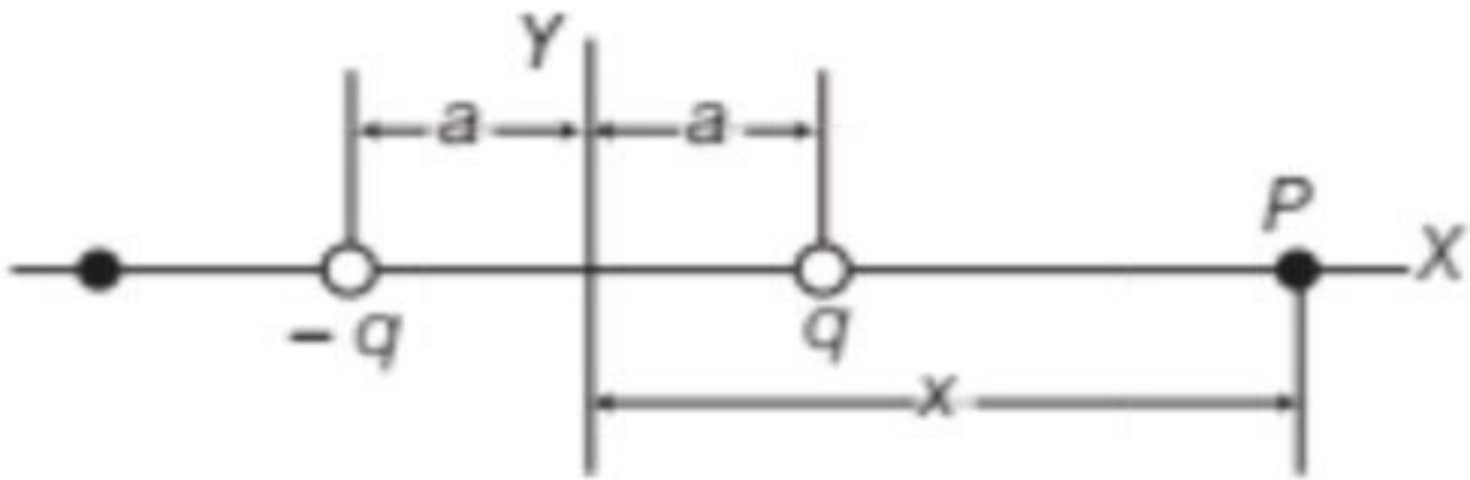
Thus, electrostatic potential at any point in the equatorial plane of dipole is zero.

Differences between electric potential due to an electric dipole and due to a single charge are given as below:

(i) The potential due to a dipole depends not just on r but also on the angle between the position vector **r** and dipole moment vector **p**.

(ii) The electric potential due to dipole falls off at large distance as $1/r^2$ not as $1/r$, which is a characteristic of the potential due to single charge.

EXAMPLE |9| An electric dipole consists of two charges of equal magnitude and opposite signs separated by a distance $2a$ as shown in figure. The dipole is along the X-axis and is centred at the origin.



(i) Calculate the electric potential at point P.

(ii) Calculate V at a point far from the dipole.

Sol. (i) For the point P in figure,

$$V = k_e \Sigma \frac{q_i}{r_1} = k_e \left(\frac{q}{x-a} - \frac{q}{x+a} \right) = \frac{2k_e qa}{x^2 - a^2}$$

(ii) If point P is far from the dipole, such that $x >> a$, then a^2 can be neglected in the terms, $x^2 - a^2$ and V becomes

$$V = \frac{2k_e qa}{x^2} \qquad [\because x >> a]$$

EQUIPOTENTIAL SURFACES

Any surface which has same electrostatic potential at every point, on it is called an equipotential surface

The shape of equipotential surface due to a

(i) line charge is cylindrical (ii) point charge is spherical

Different properties of equipotential surfaces are given as below:

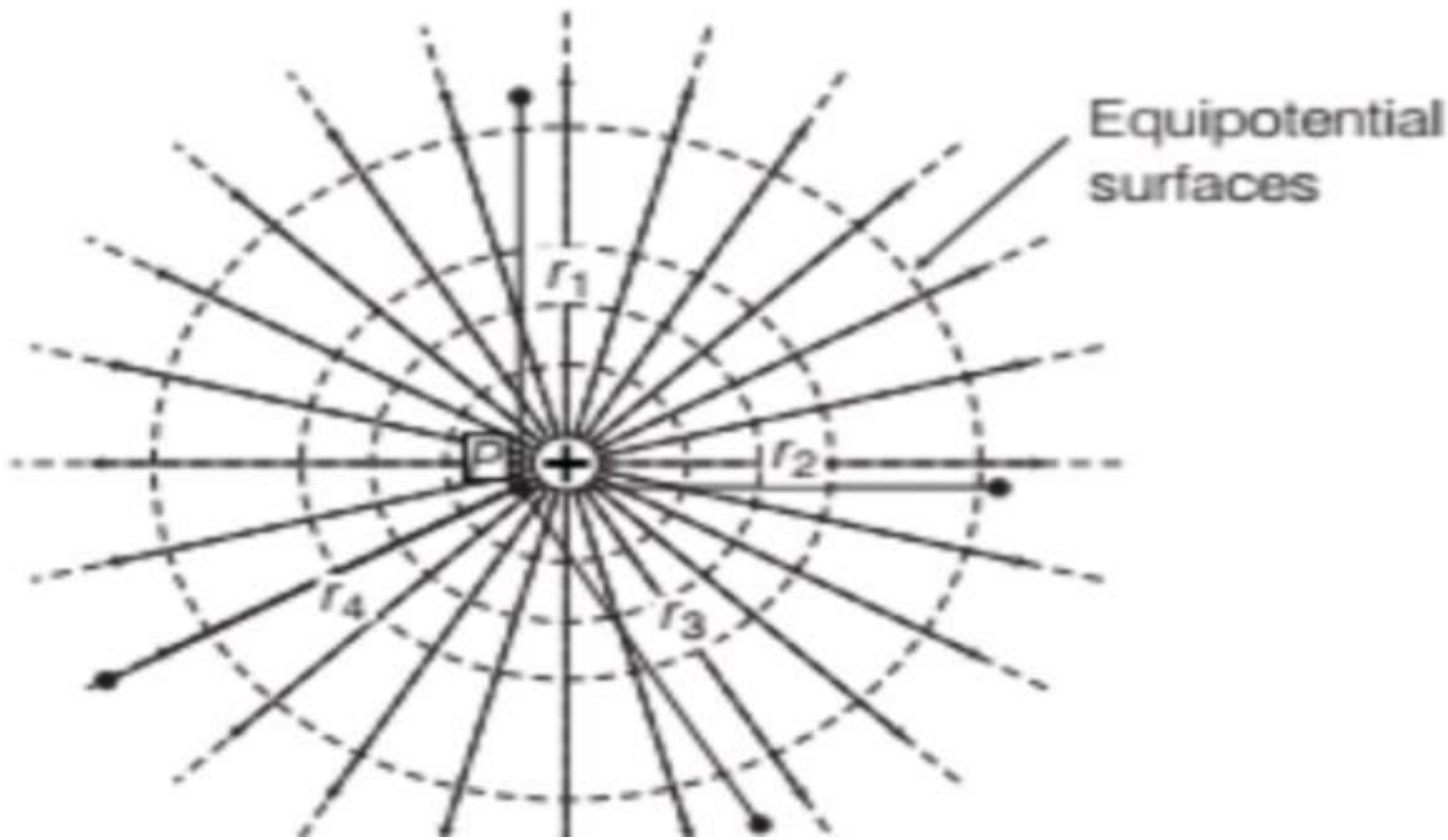
(i) Equipotential surfaces do not intersect each other as it gives two directions of electric field at intersecting point which is not possible.

(ii) Equipotential surfaces are closely spaced in the region of strong electric field and widely spaced in the region of weak electric field

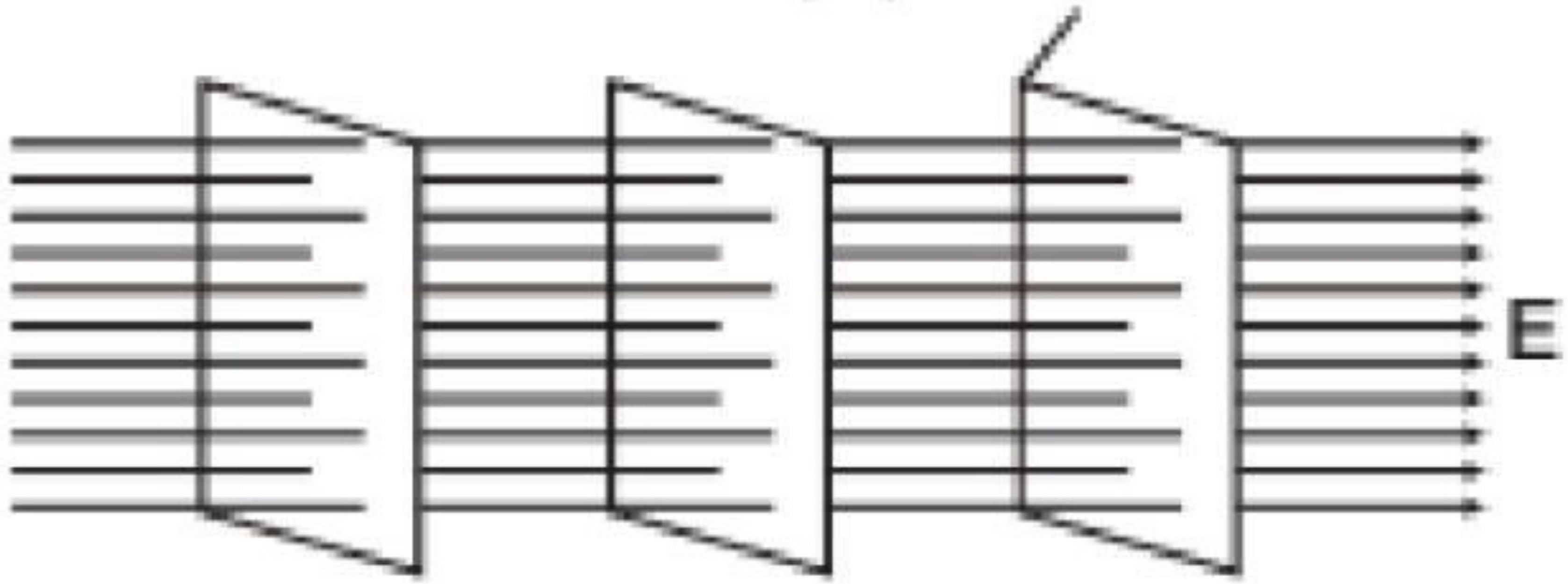
- **iii) For any charge configuration, equipotential surface through a point is normal to the electric field at that point and directed from one equipotential surface at higher potential to the other equipotential surface at lower potential.**
- **(iv) No work is required to move a test charge on anequipotential surface.**
- **(v) For a uniform electric field E, let along X-axis, the equipotential surfaces are normal to the X-axis, i.e. planes parallel to the YZ-plane.**

Equipotential Surfaces in Different Cases

Case I The equipotential surfaces produced by a point charge or a spherically symmetrical charge distribution is a family of concentric spheres as shown below in the figure.

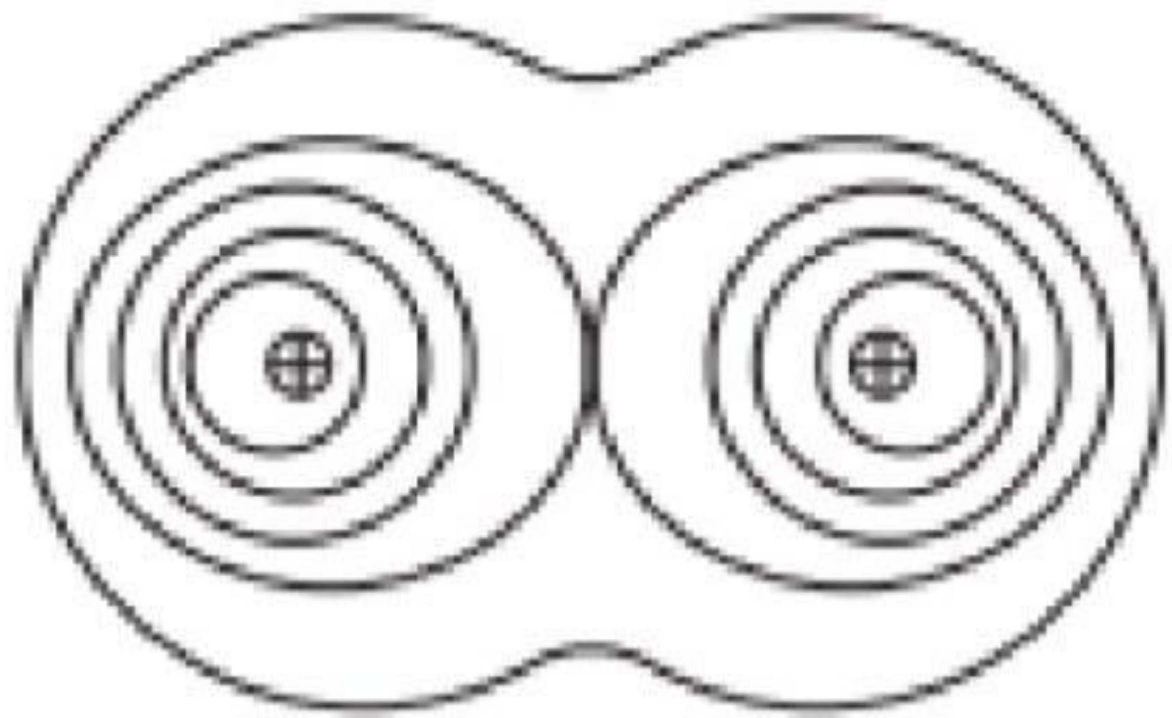


Case II The equipotential surfaces for a uniform electric field are as shown below in figure by dotted lines.



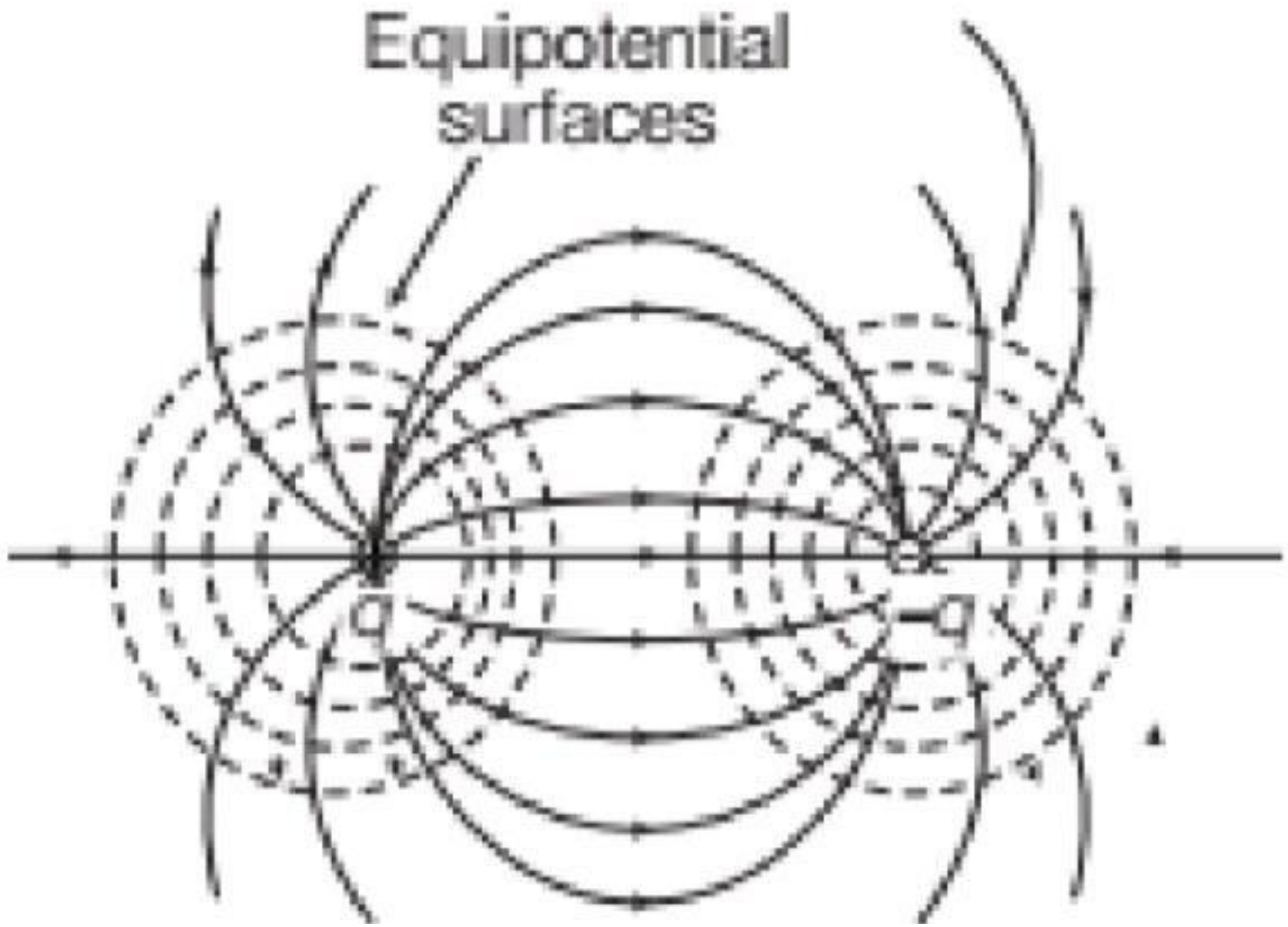
Equipotential surfaces for a uniform electric field

Case III The equipotential surfaces due to two identical positive charges are as shown below

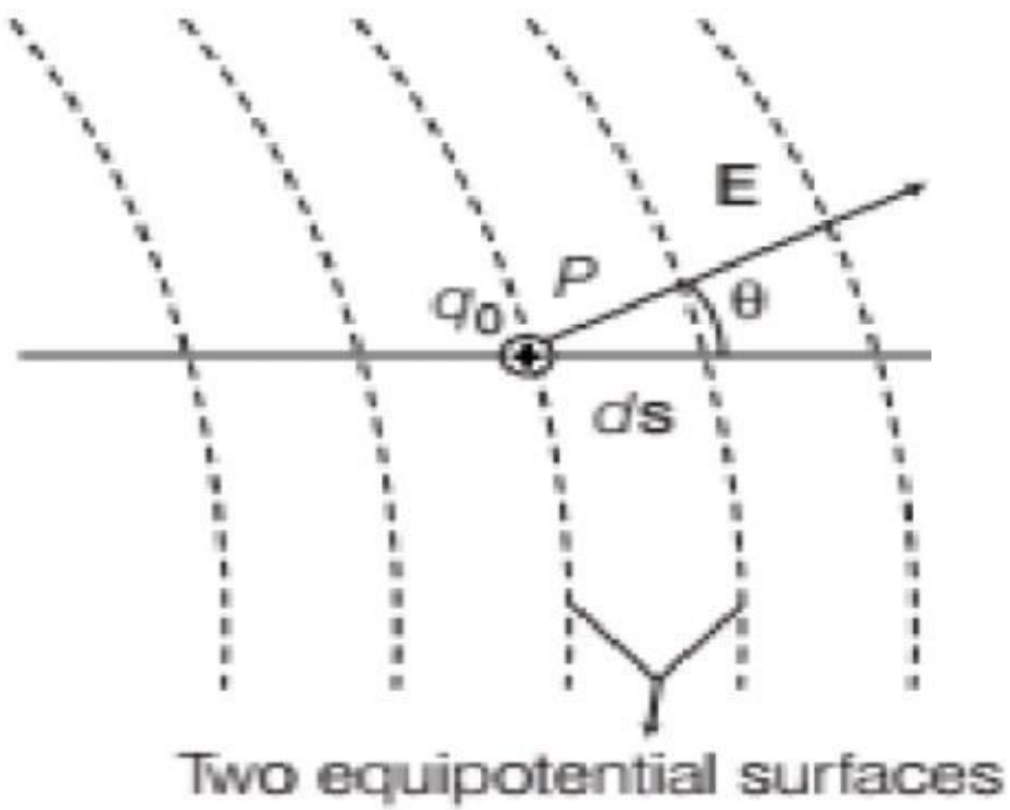


Equipotential surfaces due to two positive charges

Case IV The equipotential surfaces for an electric dipole are as shown below in the figure by dotted lines.



Relation between Electric Field and Electric Potential



Displacement of charge between two equipotential surfaces

For the simple situation in which the electric field E is uniform.

$$E = -\frac{\Delta V}{\Delta s}$$

Further, the magnitude of an electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point. This is called **potential gradient**, i.e.

$$|\mathbf{E}| = -\frac{|dV|}{ds} = -\text{(Potential gradient)}$$

We thus arrive at two important conclusions concerning the relation between electric field and potential which are as given below

(i) Electric field is in the direction in which the potential decreases steepest.

(ii) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotential surface at the point.

EXAMPLE |11| A small particle carrying a negative charge of 1.6×10^{-19} C is suspended in equilibrium between the horizontal metal plates 5 cm apart, having a potential difference of 3000 V across them. Find the mass of the particle.

Sol. Here, $q = -1.6 \times 10^{-19}$ C,

$$dr = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$$

and $dV = 3000\text{V}$

$$\therefore \quad E = -\frac{\partial V}{\partial r} = \frac{-3000}{5 \times 10^{-2}} = -6 \times 10^{4} \text{ Vm}^{-1}$$

As, the charged particle remains suspended in equilibrium, therefore

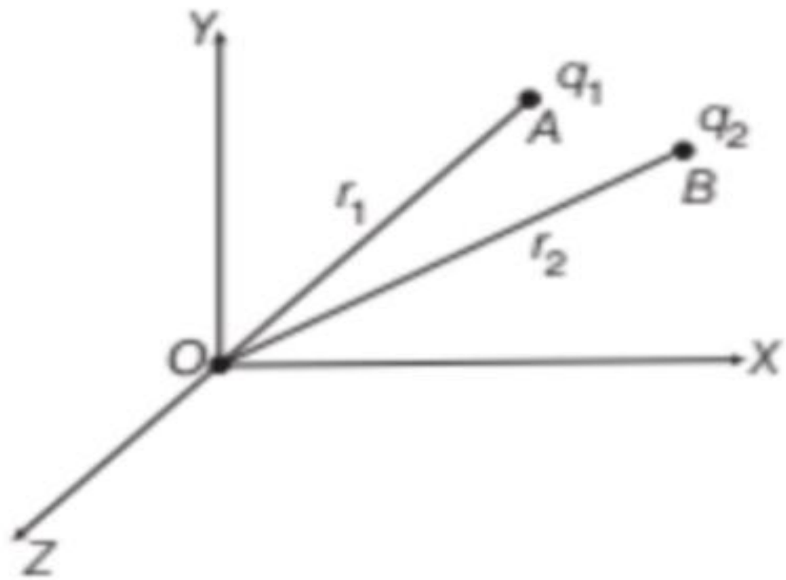
$$F = mg = qE$$

$$\therefore \quad m = \frac{qE}{g} = \frac{(-1.6 \times 10^{-19}) \times (-6 \times 10^{4})}{9.8}$$

$$= 9.8 \times 10^{-16} \text{ kg}$$

- **ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES**
- **Electrostatic potential energy of a system of point charges is defined as the total amount of work done in bringing the different charges to their respective positions from infinitely large mutual separations.**

Electrostatic Potential Energy of a System of Two Point Charges



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$$\therefore \quad V = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{AB}}$$

where, r_{AB} is the distance between points A and B.

By definition, work done in carrying charge q_2 from ∞ to B is

$$W = \text{Potential} \times \text{Charge} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1}{r_{AB}} \cdot q_2$$

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{AB}}$$

This work is stored in the system of two point charges q_1 and q_2 in the form of electrostatic potential energy U of the system.

Thus, $$\boxed{U = W = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{AB}}}$$

Electrostatic potential energy is a scalar quantity. In the above formula, the values of q_1 and q_2 must be with proper signs. If $q_1, q_2 > 0$, then potential energy is positive. It means that two charges are of same sign, i.e. they repel each other. Then, in bringing closer, work is done against the force of repulsion, so that the electrostatic potential energy of the system increases.

Conversely, in separating them, work is obtained from the system, so the potential energy of the system decreases.

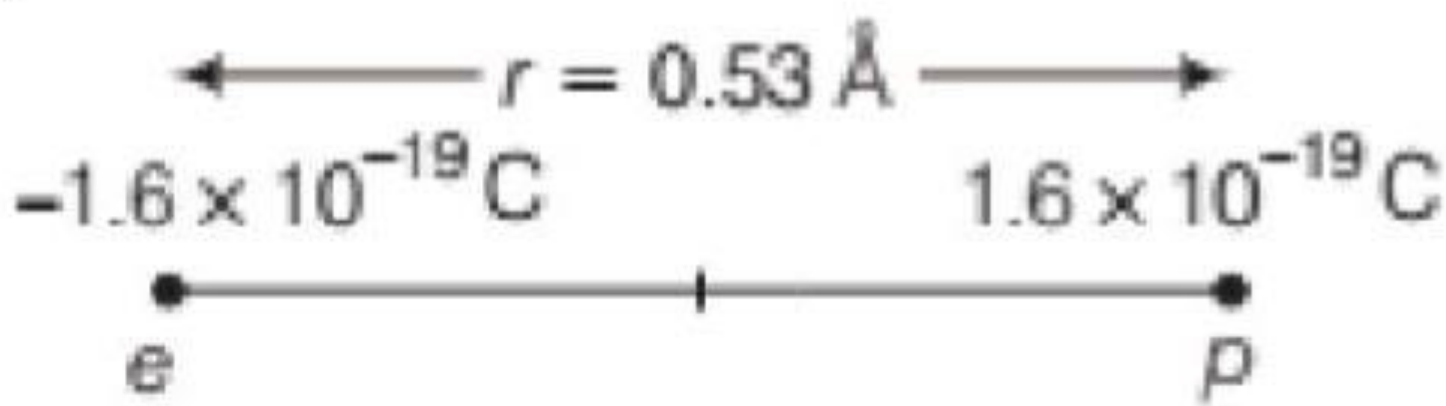
EXAMPLE 14| In a hydrogen atom, the electron and proton are bound at a distance of about 0.53 A

.(i) Estimate the potential energy of the system in eV, taking the zero of the potential energy at infinite separation of the electron from proton.

(ii) What is the minimum work required to free the electron, given that its kinetic energy in the orbit is half the magnitude of potential energy obtained in (i) ?

(iii) What are the answers to (i) and (ii) above, if the zero of potential energy is taken at 1.06 Å separation?

Sol. Charge on electron, $q_e = -1.6 \times 10^{-19}$ C and charge on proton, $q_p = 1.6 \times 10^{-19}$ C



(i) Potential energy of the system

= Potential energy at infinity

– Potential energy at a distance of 0.53 Å

$$= 0 - \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{r}$$

$$= 0 - \frac{9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{0.53 \times 10^{-10}}$$

$$= -43.47 \times 10^{-19}\ \text{J} \qquad [\because 1\ \text{eV} = 1.6 \times 10^{-19}\ \text{J}]$$

$$= -\frac{43.47 \times 10^{-19}}{1.6 \times 10^{-19}} = -\ 27.16\ \text{eV}$$

(ii) The kinetic energy $= -\frac{1}{2} \times$ Potential energy

$$= -\frac{1}{2} \times (-27.16) = 13.58 \text{ eV}$$

Total energy $= KE + PE = 13.58 - 27.16 = -13.58$ eV

Thus, the minimum work done required to free the electron is 13.58 eV.

(iii) Potential energy at separation of 1.06 A

$$= \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_e q_p}{1.06 \times 10^{-10}}$$

$$= \frac{-9 \times 10^9 \times (-1.6) \times 10^{-19} \times 1.6 \times 10^{-19}}{1.06 \times 10^{-10}}$$

$$= -21.73 \times 10^{-19} \text{ J}$$

$$= -\frac{21.73 \times 10^{-19}}{1.6 \times 10^{-19}} = -13.58 \text{ eV}$$

Thus, the potential energy of the system at 1.06 Å

= PE at distance 1.06 Å – PE at distance 0.53 Å

= – 13.58 – (– 27.16) = 13.58 eV

Thus, on shifting the zero of potential energy, work required to free electron remains same and it is equal to 13.58 eV.