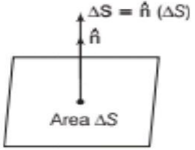
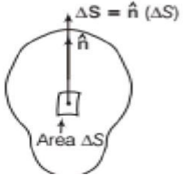
TOPIC 4

Electric Flux

AREA VECTOR

The vector associated with every area element of a closed surface is taken to be in the direction of t normal thus, the area element vector ΔS at a point on a closed surface is equal to ΔS ñ, where ΔS is the magnitude of the area element and â is a unit vector in the direction of outward normal at the point.



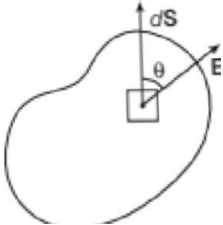


ELECTRIC FLUX

Electric flux linked with any surface is defined as the total number of electric field lines that normally pass through that surface.

$$d\phi = \mathbf{E} \cdot d\mathbf{S} = E\, dS \cos\theta$$

$$\phi = \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_S E\, dS \cos\theta$$



Electric flux is a scalar quantity. But it is a property of vector field.
SI unit of electric flux is s $N\text{-}m^2C^{-1}$
and dimensional formula of electric flux is expressed as

$$\phi = [MLT^{-2}][L^2][AT]^{-}$$

$$= [ML^3T^{-3}A^{-1}]$$

If $\oint \mathbf{E} \cdot d\mathbf{S}$ over a closed surface is negative, then the surf

encloses a net negative charge.

Special cases

(i) For $0° < \theta < 90°$, ϕ is positive.

(ii) For $\theta = 90°$, ϕ is zero.

(iii) For $90° < \theta < 180°$, ϕ is negative.

EXAMPLE |1| A box encloses an electrical dipole consisting of charge 5 μC and −5 μC and of length 10 cm. What is the total electric flux through the box?

All India 2011

Sol. Since, an electric dipole consists of two equal and opposite charges, the net charge on the dipole is zero.

Hence, the net electric flux coming out of the closed surface of the box or through the box is zero.

GAUSS' THEOREM

Statement

The surface integral of the electric field intensity over any closed surface (called Gaussian surface) in free space is equal to $\frac{1}{\varepsilon_0}$ times the net charge enclosed within the surface.

$$\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{1}{\varepsilon_0} \sum_{i=1}^{n} q_i = \frac{q}{\varepsilon_0}$$

where, $q = \sum_{i=1}^{n} q_i$ is the algebraic sum of all the charges inside the closed surface.

EXAMPLE |2| A charge q is placed at the centre of a cube of side l. What is the electric flux passing through each face of the cube? **All India 2012; Foreign 2010**

Sol. By Gauss' theorem, total electric flux linked with a closed surface is given by

$$\phi = \frac{q}{\varepsilon_0}$$

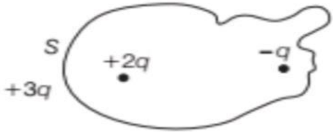
where, q is the total charge enclosed by the closed surface.

$\therefore$ Total electric flux linked with cube, $\phi = \frac{q}{\varepsilon_0}$.

As charge is at centre, therefore electric flux is symmetrically distributed through all 6 faces.

$\therefore$ Flux linked with each face $= \frac{1}{6}\phi = \frac{1}{6} \times \frac{q}{\varepsilon_0} = \frac{q}{6\varepsilon_0}$

EXAMPLE |3| Figure shows three point charges, $+2q$, $-q$ and $+3q$. Two charges $+2q$ and $-q$ are enclosed within a surface S. What is the electric flux due to this configuration through the surface S? **Delhi 2010**



Sol. Electric flux through the closed surface S is

$$\phi_S = \frac{\Sigma q}{\varepsilon_0} = \frac{+2q - q}{\varepsilon_0} = \frac{q}{\varepsilon_0}$$

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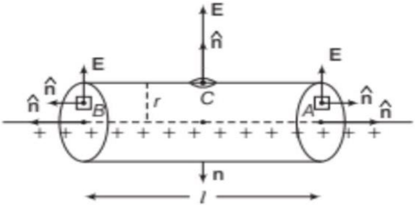
Charge $+3q$ is outside the closed surface S, therefore it would not be taken into consideration in applying Gauss' theorem.

- **Some Features of Gauss' Law**
- 1) Gauss' law is true for any closed surface, no matter what its shape or size be.
- (ii) In the situation, when the surface is so chosen that there are some charges inside and some outside, the electric field is due to all the charges, both inside and outside the closed surface.
- (iii) Gauss' law is often useful when the system has some symmetry. This is facilitated by the choice of a suitable Gaussian surface.

Applications of Gauss' Theorem

- <u>**Field due to an Infinitely Long Thin Straight Charged Wire**</u>



Cylindrical guassian surface

From Gauss' law, $\phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$

Now, $\phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = \oint \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$

$$= \oint_A \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_B \mathbf{E} \cdot \hat{\mathbf{n}}\, dS + \oint_C \mathbf{E} \cdot \hat{\mathbf{n}}\, dS$$

$$\therefore \oint_S \mathbf{E} \cdot d\mathbf{S} = \oint_A \mathbf{E}\, dS \cos 90° + \oint_B \mathbf{E}\, dS \cos 90° + \oint_C \mathbf{E}\, dS \cos 0°$$

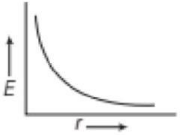
$$= \oint_C E\, dS = E(2\pi r l)$$

Charge enclosed in the cylinder, $q = \lambda l$

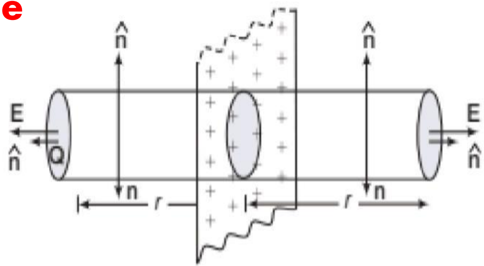
$$\therefore \quad E(2\pi r l) = \frac{\lambda l}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{\lambda}{2\pi\varepsilon_0 r}}$$

Vectorially, $\mathbf{E} = \dfrac{\lambda}{2\pi\varepsilon_0 r}\hat{\mathbf{r}}$

E VS r GRAPH



Field due to a Thin Infinite PlaneSheet of Charge



Flux through curved surface is = 0

Because ϴ=90

Flux through the flat surfaces = EA+ EA = 2EA.

. Total electric flux over the entire surface of cylinder,=2EA

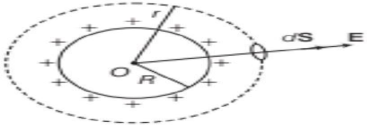
Total charge enclosed by the cylinder, $q = \sigma A$

According to Gauss' law, $\phi_E = \dfrac{q}{\varepsilon_0}$

$$\therefore \qquad 2EA = \frac{\sigma A}{\varepsilon_0} \quad \text{or} \quad \boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

Field due to a Uniformly ChargedThinSpherical Spherical Shell

Let σ be the uniform surface charge density of a thin spherical shell of radius (R).



(i) **At a point outside the shell ($r > R$)**

Since, **E** and $d\mathbf{S}$ are in the same direction.

$$\therefore \quad \phi_E = \oint_S \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0} \text{ or } E(4\pi r^2) = \frac{q}{\varepsilon_0}$$

$$\therefore \quad \boxed{E = \frac{q}{4\pi\varepsilon_0 r^2}}$$

Since, $$q = \sigma \times 4\pi R^2$$

$$\therefore \quad E = \frac{\sigma R^2}{\varepsilon_0 r^2}$$

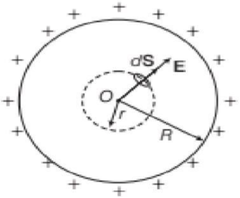
Vectorially, $$\mathbf{E} = \frac{\sigma R^2}{\varepsilon_0 r^2} \hat{\mathbf{r}}$$

(ii) **At a point on the surface of the shell** $(r = R)$

$$E = \frac{q}{4\pi\varepsilon_0 R^2}$$

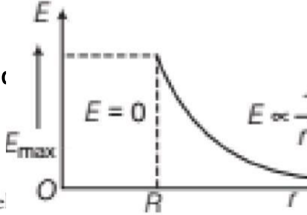
and $$E = \frac{\sigma}{\varepsilon_0}$$

(iii) **At a point inside the shell** $(r < R)$



Q=0

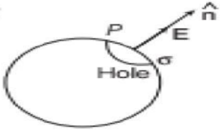
Hence E=(



Here, the charge inside the Gaussian surface she

EXAMPLE |4| A hollow charged conductor has a tiny hole cut into its surface. Show that the electric field in the hole is $\left(\frac{\sigma}{2\varepsilon_0}\right)\hat{\mathbf{n}}$, where $\hat{\mathbf{n}}$ is the unit vector in the outward normal direction and σ is the surface charge density near the hole. NCERT

Sol. Surface charge density near the hole = σ



Unit vector = $\hat{\mathbf{n}}$ (normal directed outwards)

Let P be the point on the hole. The electric field at point P closed to the surface to conductor, according to Gauss' theorem,

$$\oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}$$

where, q is the charge near the hole.

$$EdS\cos\theta = \frac{\sigma dS}{\varepsilon_0}$$

$[\because \sigma = q/dS \Rightarrow q = \sigma dS$, where dS = area]

$\therefore$ Angle between electric field and area vector is 0°.

$$\therefore \quad E\,dS = \frac{\sigma\,dS}{\varepsilon_0} \qquad [\because \cos 0^\circ = 1]$$

$$\Rightarrow \quad E = \frac{\sigma}{\varepsilon_0} \Rightarrow \mathbf{E} = \frac{\sigma}{\varepsilon_0}\,\hat{\mathbf{n}}$$

This electric field is due to the filled up hole and the field due to the rest of the charged conductor. The two fields inside the conductor are equal and opposite.

So, there is no electric field inside the conductor. Outside the conductor, the electric fields are equal in the same direction.

So, the electric field at point P due to each part

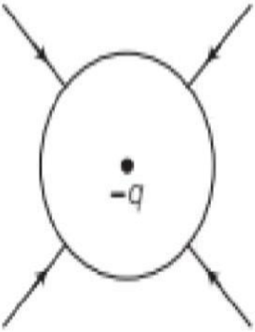
$$= \frac{1}{2}\,\mathbf{E} = \frac{\sigma}{2\varepsilon_0}\,\hat{\mathbf{n}}$$

EXAMPLE |5| A point charge causes an electric flux $-3\times10^{-14}\ \text{N-m}^2/\text{C}$ to pass through a spherical Gaussian surface.

(i) Calculate the value of the point charge.

(ii) If the radius of the Gaussian surface is doubled, how much flux would pass through the surface? **Foreign 2008**

Sol. (i) By Gauss' theorem, total electric flux through closed Gaussian surface is given by



$$\phi = \frac{q}{\varepsilon_0}$$

$$\therefore \quad q = \phi\varepsilon_0$$

But electric flux passing through the surface,

$$\phi = -3 \times 10^{-14} \text{ N-m}^2/\text{C}$$

$$\therefore \quad q = (-3 \times 10^{-14}) \times 8.85 \times 10^{-12} = -26.55 \times 10^{-26}\,\text{C}$$
$$= -2.655 \times 10^{-25}\,\text{C}$$

(ii) Electric flux passing through the surface remains unchanged because it depends only on charge enclosed by the surface and is independent of its size.

Note Electric flux, Gauss's law and numericals based on them have been frequently asked in previous years 2015, 2014, 2013, 2012, 2011, 2010.