Volume charge density, $\delta = \dfrac{q}{\frac{4}{3}\pi R^3}$

$$= \frac{79 \times 1.6 \times 10^{-19}}{4/3 \times 3.14 \times (7.0 \times 10^{-15})^3}$$

$$= 0.088 \times 10^{26}$$

$$= 8.8 \times 10^{24}\ \mathrm{Cm^{-3}}$$

- # ELECTRIC FIELD

The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point.

The charge Q which produces the electric field is called source charge and the charge q which experiences the effect of source charge is called **test charge.**

• Electric Field Intensity

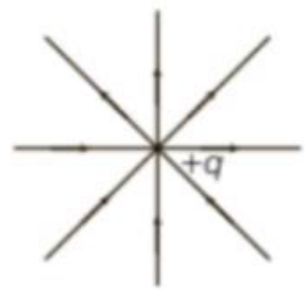
The electric field intensity at any point due to source charge is defined as the force experienced per unit positive test charge placed at that point without disturbing the source charge.

It is expressed as, $E = F/q_0$

where, E = electric field intensity and F = force experienced by the test charge q_0

It is a vector quantity and its SI unit is NC^{-1}

The figure (a) is representing the electric field due to charge+q. In this, it can be seen that for a positive charge, the electric field vector is directed radially outwards, i.e. away from positive charge.



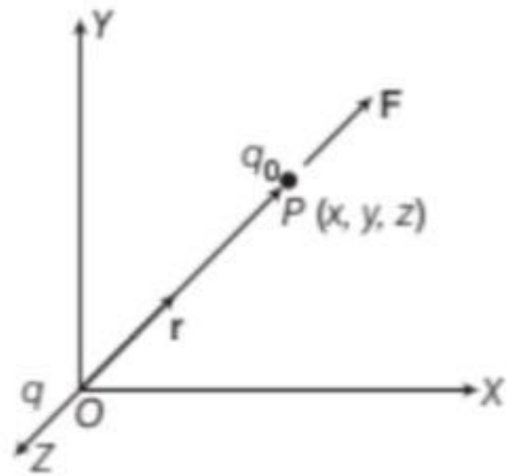
(a)

The figure (b) is representing the electric field due to charge -q. In this, it can be seen that for a negative charge, the electric field vector is directed radially inwards, i.e. towards negative charge.

(b)

•Electric Field due to a Point Charge

We have to find the electric field at a point P due to a point charge + q placed at the origin such that OP = r.



Electric fi n coordinate frame

- To find the electric field at point P, we have to find the electric force on a test charge q, placed at point P, due to source charge q.

According to Coulomb's law, force on the test charge o due to charge q is given by

$$\mathbf{F} = \frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathbf{r}}$$

If E is the electric field at a point P, then

$$\mathbf{E} = \lim_{q_0 \to 0}\frac{\mathbf{F}}{q_0} = \lim_{q_0 \to 0}\left(\frac{1}{q_0}\cdot\frac{1}{4\pi\varepsilon_0}\frac{qq_0}{r^2}\hat{\mathbf{r}}\right)$$

$$\Rightarrow \qquad \mathbf{E} = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}\hat{\mathbf{r}} \qquad \ldots(\mathrm{i})$$

The magnitude of the electric field at a point P is given by

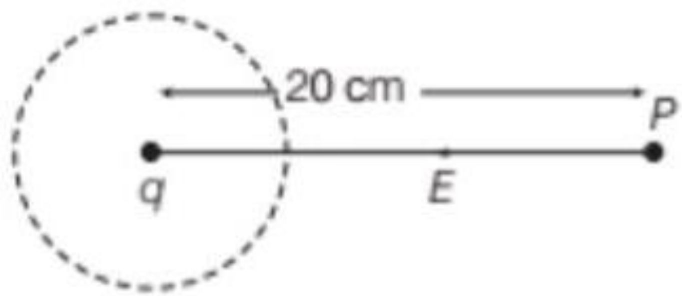
$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q}{r^2}}$$

- From the above formula, it is clear that electric field at any point in space due to a charge depends only on the distance. That means, the magnitude of electric field due to point charge is same at all the points of sphere, i.e. it has spherical symmetry.

- **EXAMPLE 18** A conducting sphere of radius 10 cm hasan unknown charge. If the electric field 20 cm from the centre of the sphere is 1.5 x 10^3 N/C and points radially inwards, then what is the net charge on the sphere?

  Sol. Let the value of unknown charge be q.

  Electric field at 20 cm away, E = 15×10^3 N/C



From the formula, electric field,

$$E = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q}{r^2}$$

$$\Rightarrow 1.5 \times 10^3 = \frac{9 \times 10^9 \times q}{(20 \times 10^{-2})^2}$$

$$\therefore \quad q = \frac{1.5 \times 10^3 \times 20 \times 20 \times 10^{-4}}{9 \times 10^9} = 6.67 \times 10^{-9} C$$

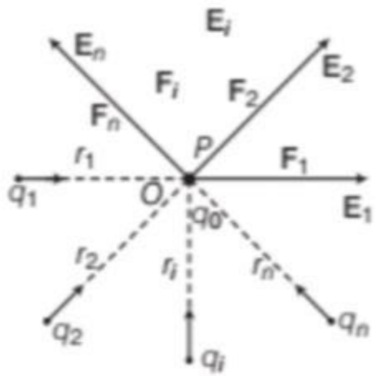
As the electric field is radially inwards which shows thatthe nature of unknown charge q is negative.

•Electric Field due to System of Charges

Consider that point charges q_1, q_2, q_3,..., q_n exert forces F_1 ,F_2 ,F_3,..., F_n on a test charge q_0 placed at origin O. Let F_i be the force due to ith charge q_i on q_o, then

$$\mathbf{F}_i = \frac{1}{4\pi\varepsilon_0}\frac{q_i q_0}{r_i^2}\hat{\mathbf{r}}_i$$

where, r, is the distance of the test charge 40 from q;.



The electric field at the observation point P is given by

$$\mathbf{E}_i = \lim_{q_0 \to 0}\frac{\mathbf{F}_i}{q_0} = \lim_{q_0 \to 0}\left[\frac{1}{q_0}\left(\frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i q_0}{r_i^2}\hat{\mathbf{r}}_i\right)\right]$$

$$\mathbf{E}_i = \frac{1}{4\pi\varepsilon_0}\cdot\frac{q_i}{r_i^2}\hat{\mathbf{r}}_i \qquad \ldots(i)$$

If E is electric field at point P due to the system of charges, then by principle of superposition of electric fields,

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2 + \mathbf{E}_3 + \cdots + \mathbf{E}_n = \sum_{i=1}^{n} \mathbf{E}_i$$

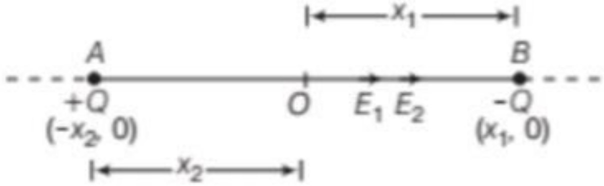
Using Eq. (i), we get

$$\mathbf{E} = \sum_{i=1}^{n} \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i$$

or $$\boxed{\mathbf{E} = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{r_i^2} \hat{\mathbf{r}}_i}$$

- **EXAMPLE 9** Two charges +Q and -Qare kept at points (-x2,0) and (x1,0) respectively, in the XY-plane. Find the magnitude and direction of the net electric field at the origin (0,0).
- **Hints**: To find the electric field intensity at a point due to two charges, first of all find the individual electric field due to both charges and then find the resultant field by using vector addition.

Sol



Electric field intensity at point O due to +Q charge,

$$E_1 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} \text{ (towards } B\text{)} \qquad \text{...(i)}$$

Electric field intensity at point O due to -Q charge,

$$E_2 = \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \text{ (towards } B\text{)} \qquad \text{...(ii)}$$

E_1 and E_2 act along the same direction.

Net electric field intensity at point O is given by

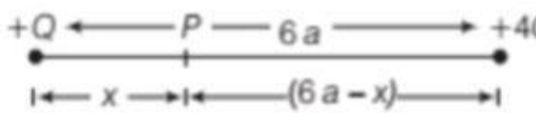
$$E = E_1 + E_2$$

$$= \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_2)^2} + \frac{1}{4\pi\varepsilon_0} \times \frac{Q}{(x_1)^2} \text{ (towards} B)$$

$$= \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{x_2^2} + \frac{1}{x_1^2}\right]$$

EXAMPLE [10] Two point charges + Q and +4Q are separated by a distance of 6a. Find the point on the line joining the two charges, where the electric field is zero.

Sol. The electric field is zero at point P only, if the field due to charge + Q balances the field due to charge + 4Q.



$$\therefore \frac{1}{4\pi\varepsilon_0} \cdot \frac{Q}{x^2} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{4Q}{(6a - x)^2} \Rightarrow \frac{1}{x^2} = \frac{4}{(6a - x)^2}$$

$$\Rightarrow \quad \frac{1}{x} = \frac{2}{(6a - x)}$$

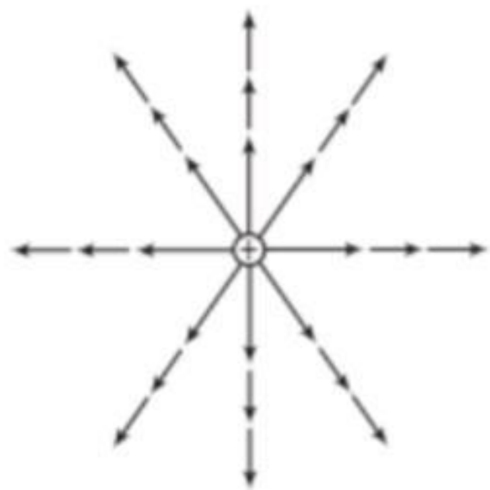
$$\Rightarrow \quad 2x = 6a - x$$

$$\Rightarrow \quad x = 2a$$

The required point is at a distance of 2a from + Q.

• ELECTRIC FIELD LINES

- An electric field line in general is a curve drawn in such a way that the tangent to it at each point is in the direction of the electric field at that point. A field line is a space curve, i.c. a curve in three dimensions.Electric field lines are thus used to pictorially map the electric field around a charge or a configuration of charges.



Field lines showing electric field of a point charge

- The density of field lines is more near the charge. Away from the charge, the field is weak, so the density of fieldlines is less.

•Properties of Electric Field Lines

Electric field lines follow some important properties whichare discussed below

(i) Electric field lines start from positive charges and end at negative charges. In the case of a single charge, they may start or end at infinity.

(ii) Tangent to any point on electric field lines shows the direction of electric field at that point.

(iii) Two field lines can never intersect each other because if they intersect, then two tangents drawn at that point will represent two directions of field at that point, which is not possible.

(iv) In a charge free region, electric field lines can be takento be continuous curves without any breaks.

(v) Electric field lines do not form closed loops (becauseof conservative nature of electric field).

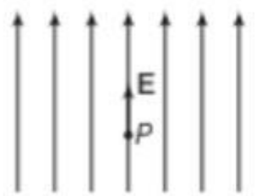
(vi) Electric field lines are perpendicular to the surface of a charged conductor.

(vii) Electric field lines contract lengthwise to represent attraction between two unlike charges.

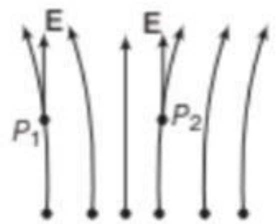
(viii) Electric field lines exert sideways pressure to representrepulsion between two like charges.

Representations of Electric Field

For different types of electric field, lines are represented as

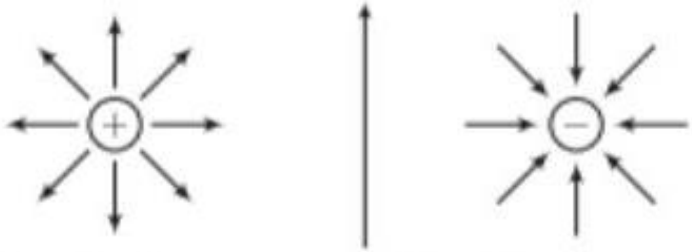


Electric field lines for a uniform field

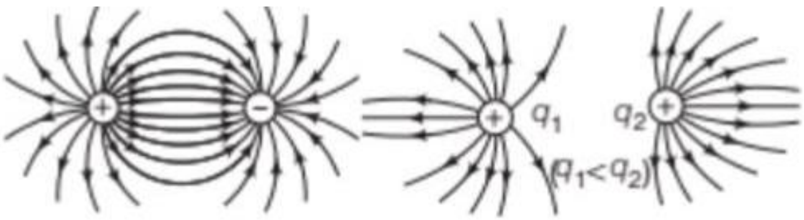


Electric field lines for a non-uniform field

The electric field lines start from positive charges and end at negative charges.



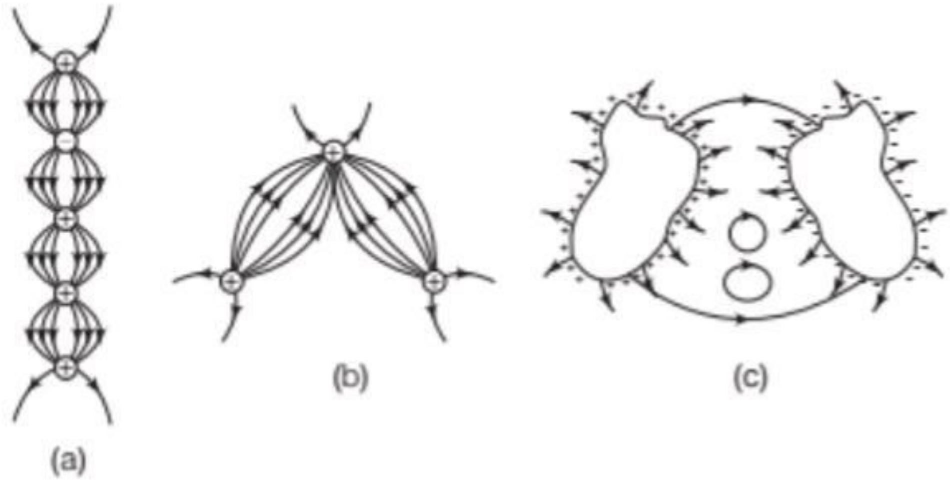
Direction is away from the positive charge   Direction is towards the negative charge



(a)   (b)

It is a common misconception that the path traced by a positive charge is a field line. The path traced by a unit positive test charge represents a field line only when it moves along a straight line.

EXAMPLE |11| Explain, why the following curves cannot possibly represent electrostatic field lines?



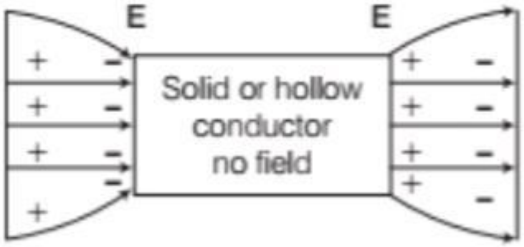
Sol (a) Electrostatic field lines cannot start from a negativecharge.

(b) Electrostatic field lines cannot end at positive charge.

(c) Electrostatic field lines cannot form closed loops.

• Conductors in an Electrostatic Field

(i) Electric field lines do not pass through a conductor. Hence, the interior of the conductor is free from the influence of the electric field.



Electrostatic shielding

(ii) Total charge of a charged conductor lies at the outer surface of the conductor.
(iii) The magnitude of field strength at any point on the surface of the conductor is proportional to surface charge density at that point.