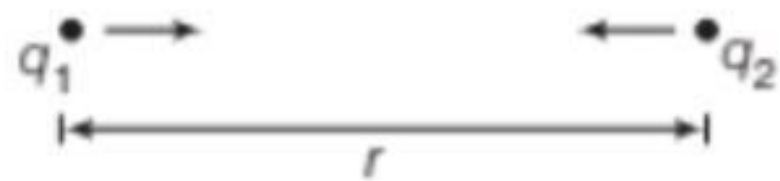
Coulomb's Law and Electrostatic Field

• COULOMB'S LAW

- The force of interaction (attraction or repulsion) between permittivity of free space. two stationary point charges in vacuum is directly proportional to the product of the charges and inversely proportional to the square of distance between them. Mathematically, electrostatic force between two stationary charges is given by

$$F = \frac{k|q_1 q_2|}{r^2}$$



where, k is a proportionality constant.

•In SI unit, k is given by

$$k = \frac{1}{4\pi\varepsilon_0}$$

$$= 9\times 10^9 \text{ N-m}^2\text{C}^{-2}$$

- where, ε_0=8.85 x 10^{-12} $C^2N^{-1}m^{-2}$ and is called the **permittivity of free space.**
- I.e, $F = 9\times 10^9 \frac{|q_1 q_2|}{r^2}$
- The Coulomb force acts along the straight line connecting the points of location of the charges. It is central and spherically symmetric.
- If $q_1 = q_2 = 1\text{C}$
- And $r = 1\text{ m}$

- Then

$$F = 9 \times 10^9 \frac{1 \times 1}{(1)^2}$$

$$\boxed{F = 9 \times 10^9 \text{ N}}$$

- i.e. **One coulomb** is the charge, that when placed at distance of 1m from another charge of same magnitude in vacuum, experiences an electric force of repulsion of magnitude 9×10^{9} N Coulomb is a bigger unit, in practice we use smaller units like mC or µC.

Absolute Permittivity of Medium (Dielectric Constant)

- The force between two charges q_1, and q_2 located distance r apart in a medium may be expressed as,
- $$F_{medium} = \frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}$$

where, & is absolute permittivity of the medium.

Now,

$$\frac{F_{vacuum}}{F_{medium}} = \frac{\frac{1}{4\pi\varepsilon_0}\frac{|q_1 q_2|}{r^2}}{\frac{1}{4\pi\varepsilon}\frac{|q_1 q_2|}{r^2}} = \frac{\varepsilon}{\varepsilon_0}$$

- The ratio $\varepsilon/\varepsilon_0$ is denoted by ε_r, which is called **relative permittivity** of the medium with respect to vacuum. It is also denoted by K called **dielectric constant** of the medium. It has no unit being a ratio.

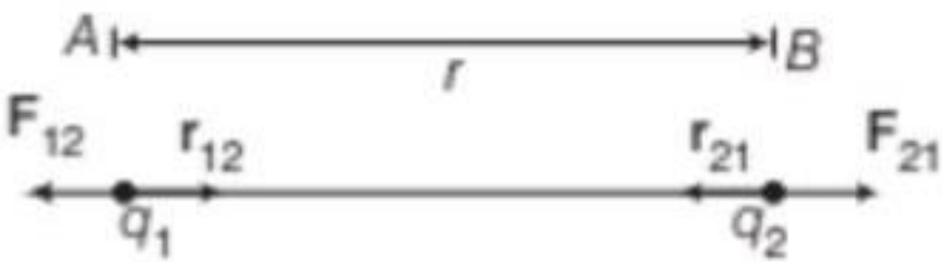
$$\therefore \quad K \text{ (or } \varepsilon_r) = \frac{\varepsilon}{\varepsilon_0} = \frac{F_{\text{vacuum}}}{F_{\text{medium}}}$$

$$\Rightarrow \quad \varepsilon = K\varepsilon_0$$

$$\therefore \quad \boxed{F_{\text{medium}} = \frac{1}{4\pi\varepsilon_0} \frac{|q_1 q_2|}{Kr^2}}$$

Coulomb's Law in Vector Form

- Consider two like charges q_1 and q_2 present at points A an B respectively in vacuum at a distance r apart.



Coulomb force between two charges

- According to Coulomb's law, the magnitude of force on charge $q_1$due to q_2 (or on charge q_2 due to q_1) is given by

$$F_{12} = F_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \quad \ldots\text{(i)}$$

- Let r_{12} be the unit vector pointing from charge q_1 to q_2

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{21} \qquad \ldots(ii)$$

[F_{12} is along the direction of unit vector r_{21}]

- Also, r_{21} be the unit vector pointing from charge q_2 to q_1.

$$F_{21} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad \ldots(iii)$$

[F_{21} is along the direction of unit vector r_{12}]

$$\therefore \qquad \bar{r}_{21} = -\bar{r}_{12}$$

- Eq. (ii) becomes,

$$F_{12} = \frac{-1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2} \hat{r}_{12} \qquad \ldots(iv)$$

- On comparing Eq. (iii) with Eq. (iv), we get

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$$F_{12} = -F_{21}$$

i.e. Coulomb's law agrees with Newton's third law.

• Comparison of Coulomb's Law with Gravitational Law

- Both the Coulomb's and Newton's law follow inverse square law. According to Newton's universal law of gravitation, "every body in the universe attracts every other body with a force which is directly proportional to the product of the masses of two bodies and inversely proportional to the square of distance between them." i.e.

$$F = \frac{Gm_1m_2}{r^2}$$

- As discussed earlier, according to Coulomb's law

$$F = \frac{kq_1q_2}{r^2}$$

- The electric force is much stronger than the gravitational force between two electrons.

$$F_E = 10^{39} F_G$$

- **EXAMPLE |1|** What is the force between two small charged spheres having charges of 2×10^{-7} C and 3×10^{-7} C placed 30 cm apart in air?
- **Sol** Given, $a_1 = 2\times10^{-7}$ C

$q_2 = 3 \times 10^{-7}$C.

r=30 cm

$= 30 \times 10^{-2}$ m

= 0.3 m

$k = 9 \times 10^9$ N - m^2 C^{-2} .

F =?

We have, $F = \frac{1}{4\pi\varepsilon_0}\frac{|q_1q_2|}{r^2}$

$$= \frac{k|q_1q_2|}{r^2} \qquad \left[\because \frac{1}{4\pi\varepsilon_0} = k\right] \quad ...(i)$$

- Substituting the given values in Eq. (i), we get

$$F = \frac{(9\times10^{9}\,\text{N-m}^2\text{C}^{-2})(2\times10^{-7}\,\text{C})\times(3\times10^{-7}\,\text{C})}{(0.3\text{m})^2}$$

$$\therefore \quad F = 6\times10^{-3}\,\text{N}$$

- This force is repulsive, since the spheres have same charges.

- **EXAMPLE |2|** The sum of two point charges is 7μC .They repel each other with a force of 1 N when kept 30 cm apart in free space. Calculate the value of each charge.

- **Sol**. Let one of two charges be xμC. Therefore, other charge will be (7 - x) μC By Coulomb's law,

$$F = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r^2}$$

$$\Rightarrow \quad 1 = 9\times 10^9 \times \frac{(x\times10^{-6})(7-x)\times 10^{-6}}{(0.3)^2}$$

$$\Rightarrow \quad 9\times10^{-2} = 9\times10^{9-12}\, x(7-x)$$

$$\Rightarrow \quad 10 = x(7-x)$$

$$\Rightarrow \quad x^2 - 7x + 10 = 0$$

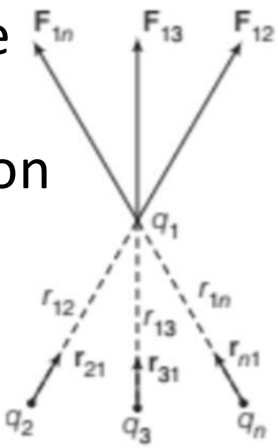
$$\Rightarrow \quad (x-2)(x-5) = 0$$

$$\therefore \quad x = 2\,\mu C \quad \text{or} \quad 5\,\mu C$$

Therefore, charges are 2μC and 5μC.

• **FORCES BETWEEN MULTIPLE CHARGES**: SUPERPOSITION PRINCIPLE

- According to the superposition principle, forces on any charge due to number of other charges is the vector sum of all the forces on that charge due to other charges, taken one at a time. The individual forces are unaffected due to the presence of other charges.
- Consider a system of n point charges $q_1, q_2, \ldots\ldots, q_3, \ldots\ldots.. q_n$ be distributed in space in a discrete manner. The charges are interacting with each other. Let the charges be $q_2, q_3, \ldots, q_n$ exert forces F_{12}, F_{13} F_{1n} respectively on charge



- Then, according to the principle of superposition, the total force on charge q_1 is given by $F_1 = F_{12} + F_{13} + \ldots + F_{1n} \ldots$(i)

- If the distance between the charges q_1 and q_2 is denoted as r_{12} and $\hat{r}_{21}$ is unit vector from charge q_2 to q_1,then

$$F_{12} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21}$$

- Similarly, the force on charge q_1 due to other charges is given by

$$F_{13} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_3}{r_{31}^2} \hat{\mathbf{r}}_{31}, \quad F_{1n} = \frac{1}{4\pi\varepsilon_0} \cdot \frac{q_1 q_n}{r_{n1}^2} \hat{\mathbf{r}}_{n1}$$

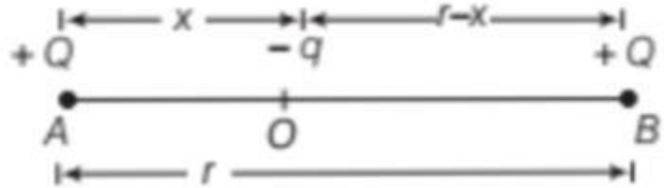
- Substituting these values in Eq.(i), we get

$$F_1 = \frac{1}{4\pi\varepsilon_0} \left(\frac{q_1 q_2}{r_{21}^2} \hat{\mathbf{r}}_{21} + \frac{q_1 q_3}{r_{31}^2} \hat{\mathbf{r}}_{31} + \cdots + \frac{q_1 q_n}{r_{n1}^2} \hat{\mathbf{r}}_{n1} \right)$$

$$\Rightarrow \qquad F_{1i} = \frac{q_1}{4\pi\varepsilon_0} \sum_{i=2}^{n} \frac{q_i}{r_{i1}^2} \hat{\mathbf{r}}_{i1}$$

- **Note** This force is on the charge which is to be studied due to othercharges.

- **EXAMPLE 13** Two charges each of +q Coulomb are placed along a line. A third charge -q is placed between them. At what position will the system be in equilibrium?
- **Sol**



- For charge-q to be in equilibrium, force on the charge -q at point O due to the charge +Q at point A should be equal and opposite to that due to the charge +Q at the point B. i.e

$$\frac{1}{4\pi\varepsilon_0}\frac{Qq}{x^2}=\frac{1}{4\pi\varepsilon_0}\frac{Qq}{(r-x)^2}$$

$$x^2=(r-x)^2$$

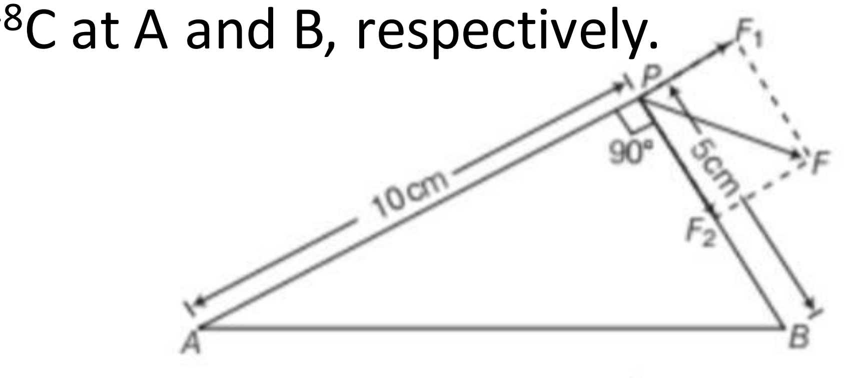
$$x^2=(r-x)$$

$$X=r/2$$

- Hence, for equilibrium the charge -q should be kept at the middle of the line joining the points A and B.

- **EXAMPLE 14** Find the magnitude of the resultant force on a charge of 1 μC held at P due to two charges of +2x 10^{-8}C and -10^{-8}C at A and B, respectively.

Given, AP = 10 cm, BP = 5cm and ZAPB = 90°,



- **Sol**. Here, charge at P. q=1μC-10^{-4}C

  Charge at A, q_1 =2x10^{-8} C .

  Charge at B, q_2=-10^{-8} C

  AP= 10cm= 0.1m, BP = 5 cm=0.05 m

  ∠APB=90°, F =?

Force at P due to q, charge at A $F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q}{AP^2}$, along

AP produced= $\frac{9\times 10^9 \times 2\times 10^{-8} \times 10^{-6}}{(0.1)^2} = 18\times 10^{-3}$N

Force at P due to q_2 charge at $F_2 = \frac{1}{4\pi\ \varepsilon_0}\frac{q_2 q}{BP^2}$, along

- PB produced = $\frac{9\times 10^{9} \times -10^{-8} \times 10^{-6}}{(0.05)^2} = -36\times 10^{-3}$ N

As, angle between F_1 and F_2 is 90°

Resultant force $F = \sqrt{F_1^2 + F_2^2} = \sqrt{(18\times 10^{-3})^2 + (-36\times 10^{-3})^2}$

$= \sqrt{(324 + 1296)\times 10^{-6}}$

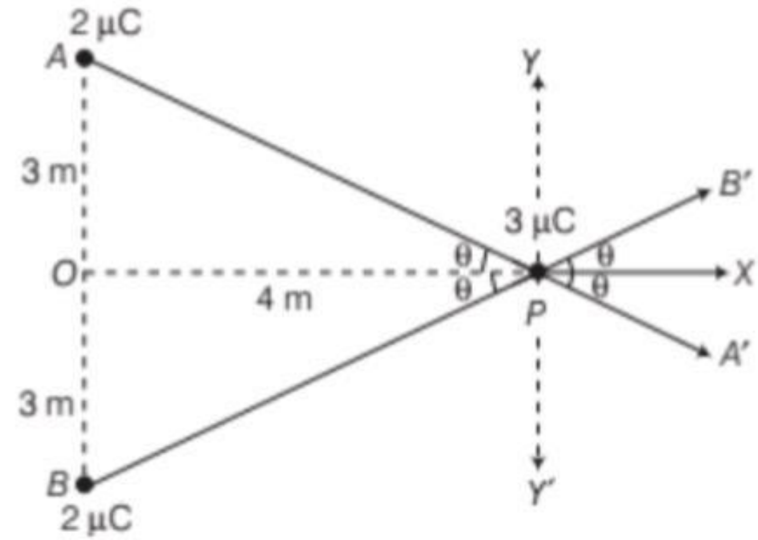
$= \sqrt{1620\times 10^{-6}}$

$= 40.2\times 10^{-3}$

$= 4.0\times 10^{-2}$ N

EXAMPLE [5] Two equal positive charges, each of 2μCinteract with a third positive charge of 3μC situated as shown in figure. Calculate the magnitude and direction of the force on the 3μC charge.

Sol. In the figure, OA = OB = 3n , OP = 4m



AP BP = √3^2 +4^2 =5m

According to Coulomb's law,

force on charge at P due to charge at A,

$$F_1 = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{AP^2}$$

$$= \frac{9\times10^9 \times(2\times10^{-6})\times(3\times10^{-6})}{5^2} = \frac{54}{25}\times10^{-3}$$

$= 2.16\times 10^{-3}$ N, along PA'.

It has two rectangular components F_2 cosΘ along PX and F_1 sinΘ along PY'.

Similarly, force on charge at P due to charge at B, F, = F

- Similarly, force on charge at P due to charge at B, $F_2 = F_1$ (in magnitude). It is along PB‘. It also has two rectangular component $F_2 \cos\Theta$ along PX and F, $\sin\Theta$ along PY. The components along PY and PY' cancel. The components along PX add up.

  Total force on 3 μC charge is

  $F = 2F_1 \cos \Theta$

  $= 2x216\times10^{-3}$ x 4/5

  $=3.5x\ 10^{-3}$ N, along PX.

• ELECTROSTATIC FORCE DUE TO CONTINUOUS CHARGE DISTRIBUTION

The region in which charges are closely spaced is said to have continuous distribution of charge. Continuous charge distribution is of three types; linear charge distribution (one dimensional), surface charge distribution (two dimensional) and volume charge distribution (three dimensional).

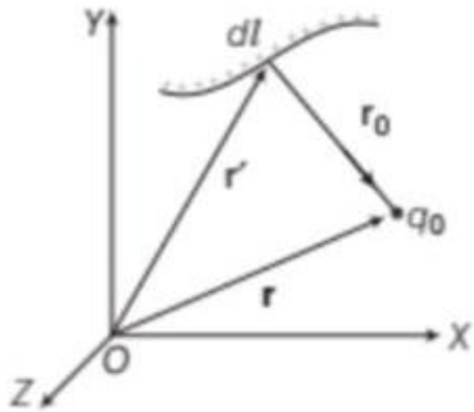
• Linear Charge Density

Linear charge density is defined as the charge per unit length of linear charge distribution. *i.e.* $\lambda = \frac{dq}{dl}$

Its SI unit is coulomb/metre.

Electric force at a point due to a linear charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0}\int_l \frac{\lambda dl}{r_0^2}\hat{\mathbf{r}}_0$$



where, r_0 = r-r', r' is the position vector of length element dl with respect to origin and r is the position vector of charge q_0 with respect to origin.

• Surface Charge Density

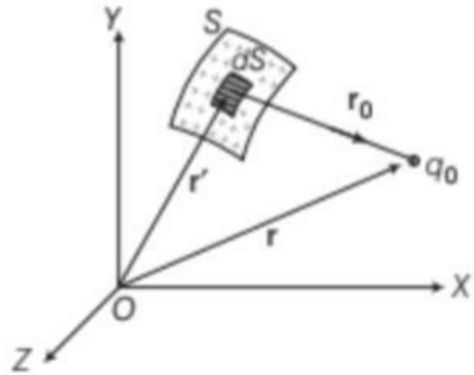
Surface charge density is defined as the charge per unit surface area of surface charge distribution.

i.e. $\lambda = \frac{dq}{dl}$

Its SI unit is coulomb/metre².

Electric force at a point due to a surface charge distribution is given by

$$\mathbf{F} = \frac{q_0}{4\pi\varepsilon_0} \int_S \frac{\sigma\, dS}{r_0^2} \hat{\mathbf{r}}_0$$



where, $r_0 = r - r'$ r' is the position vector of surface element dS with respect to origin and r is the position vector of charge q_0 with respect to origin.

• Volume Charge Density

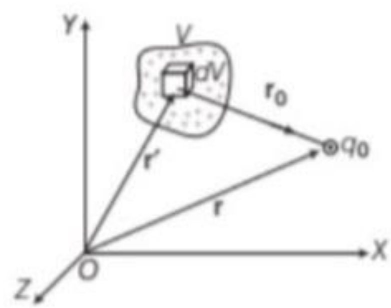
Volume charge density is defined as the charge per unitvolume of volume charge distribution.

i.e. $\delta = \frac{dq}{dV}$

Its SI unit is coulomb/metre.

Electric force at a point due to volume charge distribution is given by

$$F = \frac{q_0}{4\pi\varepsilon_0}\int_V \frac{\rho dV}{r_0^2}\hat{r}_0$$



where, $r_0 = r - r'$, r' is the position vector of volume element dV with respect to origin and r is the position vector of charge q_0 with Z respect to origin.

- **EXAMPLE 6** What charge would be required to electrify a sphere of radius 25 cm, so as to get a surface charge density of $3/\pi$ Cm^{-2} ?

Sol. Here, $r = 25$ cm $= 0.25$ m, $\sigma = \frac{3}{\pi}$ Cm^{-2}

As, $\sigma = \frac{q}{4\pi r^2}$

$\therefore$ $q = 4\pi r^2 \sigma = 4\pi \times (0.25)^2 \times \frac{3}{\pi} C = 0.75 C$

- **EXAMPLE 7** The radius of gold nucleus (Z = 79) isabout 7.0×10^{-15} m. Assuming that the positive charge is distributed uniformly throughout the nuclear volume, find the volume charge density.

Sol. The total positive charge in the nucleus is given by

$$q=+Ze=79 \times 1.6 \times 10^{-19} \text{ C}$$

Volume charge density, $\delta = \frac{q}{\frac{4}{3}\pi R^3}$

$$= \frac{79 \times 1.6 \times 10^{-19}}{4/3 \times 3.14 \times (7.0 \times 10^{-15})^3}$$

$$= 0.088 \times 10^{26}$$

$$= 8.8 \times 10^{24}\ Cm^{-3}$$

- # ELECTRIC FIELD

The electric field due to a charge Q at a point in space may be defined as the force that a unit positive charge would experience if placed at that point.

The charge Q which produces the electric field is called source charge and the charge q which experiences the effect of source charge is called **test charge.**