CUBES

When we multiply a number three times by itself, we say that the number has been cubed, and the product is called the **cube** of that number or the number raised to the power of 3. For example,

$$\text{Cube of } 1 = 1 \times 1 \times 1$$
$$= 1^3$$

$$\text{Cube of } 2 = 2 \times 2 \times 2$$
$$= 2^3$$

$$\text{Cube of } 8 = 8 \times 8 \times 8$$
$$= 8^3$$

In general, the cube of a number $x = x \times x \times x = x^3$. Look at the following table, which gives the cube of the first 15 natural numbers.

Number	Cube	Number	Cube	Number	Cube
1	1	6	216	11	1,331
2	8	7	343	12	1,728
3	27	8	512	13	2,197
4	64	9	729	14	2,744
5	125	10	1,000	15	3,375

From this table, it is clear that 1; 8; 27; 64; ... ; 3,375 are the cubes of some natural numbers. Such numbers are called **perfect cubes**. In other words, a natural number n is a perfect cube if there exists a natural number m, such that:

$$n = m \times m \times m$$
$$= m^3$$

Thus a natural number which can be expressed as a product of triplets of equal factors is known as a perfect cube.

Properties of cubes of numbers

(a) Cubes of all odd numbers are odd.

$1^3 = 1 \times 1 \times 1 = 1$

$3^3 = 3 \times 3 \times 3 = 27$

$5^3 = 5 \times 5 \times 5 = 125$

$7^3 = 7 \times 7 \times 7 = 343$

All are odd natural numbers.

(b) Cubes of all even natural numbers are even.

$2^3 = 2 \times 2 \times 2 = 8$

$4^3 = 4 \times 4 \times 4 = 64$

$10^3 = 10 \times 10 \times 10 = 1{,}000$

All are even natural numbers.

(c) Cubes of negative integers are negative.

$(-3)^3 = (-3) \times (-3) \times (-3) = -27$

$(-6)^3 = (-6) \times (-6) \times (-6) = -216$

All are negative integers.

(d) The cube of a rational number is the cube of its numerator divided by the cube of its denominator—that is, cube of $\frac{p}{q} = \left(\frac{p}{q}\right)^3 = \frac{p^3}{q^3}$.

$$\left(\frac{2}{3}\right)^3 = \frac{2^3}{3^3} = \frac{2 \times 2 \times 2}{3 \times 3 \times 3} = \frac{8}{27}$$

$$\left(\frac{5}{7}\right)^3 = \frac{5^3}{7^3} = \frac{5 \times 5 \times 5}{7 \times 7 \times 7} = \frac{125}{343}$$

(e) The product of the cubes of two numbers is equal to the cube of their products—that is, for any two natural numbers a and b, the product of their cubes is $a^3 \times b^3 = (a \times b)^3$.

CUBE ROOTS

As we know, the square root is the opposite of a square. In the same way, the **cube root** is the opposite of a cube. For example,

Cube of $8 = 8^3 = 8 \times 8 \times 8 = 512$.

Cube root of $512 = 8$.

Cube of $7 = 7^3 = 7 \times 7 \times 7 = 343$.

Cube root of $343 = 7$.

In general, the cube of $x = x \times x \times x = x^3$ and the cube root of x^3 is x. For any number x, the cube root is denoted by $\sqrt[3]{x}$.

Thus $\sqrt[3]{343} = 7$

$\sqrt[3]{216} = 6$

If the number is a negative integer, then its cube root will also be a negative integer.

$(-64)^3 = (-4)^3 = (-4) \times (-4) \times (-4)$

So $\sqrt[3]{(-64)} = (-4)$

Cube Root by Successive Subtraction

Like squares of natural numbers, cubes too have some interesting patterns.

$1^3 = 1$

$2^3 = 8 \Rightarrow 2^3 - 1^3 = 7 = 1 + 1 \times 6$

$= 1 + 2 \times 1 \times 3$

$3^3 = 27 \Rightarrow 3^3 - 2^3 = 19 = 1 + 1 \times 6 + 2 \times 6$

$= 1 + 3 \times 2 \times 3$

$4^3 = 64 \Rightarrow 4^3 - 3^3 = 37 = 1 + 1 \times 6 + 2 \times 6 + 3 \times 6$

$= 1 + 4 \times 3 \times 3$

$5^3 = 125 \Rightarrow 5^3 - 4^3 = 61$

$= 1 + 1 \times 6 + 2 \times 6 + 3 \times 6 + 4 \times 6$

$= 1 + 5 \times 4 \times 3$

$\vdots \qquad \vdots$

$9^3 = 729 \qquad 9^3 - 8^3 = 217 = 1 + 1 \times 6 + 2 \times 6 + \ldots + 8 \times 6$

$= 1 + 9 \times 8 \times 3$

Also

$1 = 1^3$

$1 + 7 = 2^3$

$1 + 7 + 19 = 3^3$

$1 + 7 + 19 + 37 = 4^3$

$\vdots \quad \vdots$

$1 + 7 + 19 + \ldots + 217 = 9^3$

Note that 2^3 is the sum of the first two numbers of the sequence 1, 7, 19, 37, ... Similarly 3^3 is the sum of first three numbers and so on. In short, these numbers (1, 7, 19, ...) may be obtained by putting $n = 1, 2, 3, \ldots$ in $1 + n(n-1) \times 3$.

Thus to find out the cube root of a given number, we go on subtracting the numbers of the sequence 1, 7, 19, 37, ... till we get a zero. The number of subtractions needed for this purpose is the cube root of the given number.

Example 1: Find the cube root of 216 by successive subtraction.

Solution: Subtract the numbers of the sequence 1, 7, 19, 37, 61, ... from 216 till we get zero from the given number.

$216 - 1 = 215$

$215 - 7 = 208$

$208 - 19 = 189$

$189 - 37 = 152$

$152 - 61 = 91$

$91 - 91 = 0$

Since we subtracted 6 times to get zero. Thus $\sqrt[3]{216} = 6$.

Example 2: Is 236 a perfect cube? If not, then what is the smallest number which you must subtract from 236 to make it a perfect cube?

Solution:

$236 - 1 = 235$

$235 - 7 = 228$

$228 - 19 = 209$

$209 - 37 = 172$

$172 - 61 = 111$

$111 - 91 = 20$

The next number to be subtracted is 127 which is greater than 20, therefore the process of subtraction does not end in zero. So 236 is not a perfect cube. If 20 is subtracted from 236, then $(236 - 20)$ is equal to 216 which is a perfect cube of 6.

Class - VIII

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Subject - Mathematics

Chapter-3. Square and Square Root; Cube and Cube Root-

Ex 3.5 (worksheet-8)

1. Find the cubes of the following numbers

(a) $2\frac{1}{5}$ (b) 1.1 (c) 0.04 (d) 7

(a) $2\frac{1}{5} = \frac{11}{5}$

$$\left(\frac{11}{5}\right)^3 = \frac{11}{5} \times \frac{11}{5} \times \frac{11}{5}$$

$$= \frac{1331}{125}$$

Ans = $\frac{1331}{125}$

(b) 1.1

$(1.1)^3 = 1.1 \times 1.1 \times 1.1$

$= 1.331$

Ans = 1.331

(c) 0.04

$(0.04)^3 = 0.04 \times 0.04 \times 0.04$

$= 0.000064$

Ans = 0.000064

(d) 7

$(7)^3 = 7 \times 7 \times 7$

$= 343$

Ans = 343.

2. Which of the following numbers are cubes of even numbers -

(a) 729 (b) 1,000 (c) 2,744 (d) 6,859

(a) 729 = odd
(b) 1,000 = even
(c) 2,744 = even
(d) 6,859 = odd

3 Which of the following are perefect cubes.
(a) 1728 (b) 2,190 (c) 18,225 (d) 9,261

(a)
2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

$1728 = \underline{2\times2\times2\times2\times2\times2}\times \underline{3\times3\times3}$

Ans. Yes, it is a perfect cube

(b)
2	2190
3	1095
5	365
73	73
	1

$2190 = 2\times3\times5\times73$

Ans = No it is not a perfect-cube

(c)
3	18225
3	6075
3	2025
3	675
3	225
3	75
5	25
5	5
	1

$18225 = 3\times3\times3\times3\times3\times3\times5\times5$

(d)
3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

$9261 = 3\times3\times3\times7\times7\times7$

Ans = Yes

4. Find the cube root of the following numbers by successive subtraction of numbers 1, 7, 9 .....

(a) 27 (b) 125 (c) 1000 (d) 1331

(a) 27

1. $27 - 1 = 26$
2. $26 - 7 = 19$
3. $19 - 19 = 0$

Ans = 3.

(b) 125

1. $125 - 1 = 124$
2. $124 - 7 = 117$
3. $117 - 19 = 98$
4. $98 - 37 = 61$
5. $61 - 61 = 0$

Ans = 5.

(c) 1000

1. $1000 - 1 = 999$
2. $999 - 7 = 992$
3. $992 - 19 = 973$
4. $973 - 37 = 936$
5. $936 - 61 = 875$
6. $875 - 91 = 784$
7. $784 - 127 = 657$
8. $657 - 169 = 488$
9. $488 - 217 = 271$
10. $271 - 271 = 0$

Ans = 10

(d). 1331

1. $1331 - 1 = 1330$
2. $1330 - 7 = 1323$
3. $1323 - 19 = 1304$
4. $1304 - 37 = 1267$
5. $1267 - 61 = 1206$
6. $1206 - 91 = 1115$
7. $1115 - 127 = 988$
8. $988 - 169 = 819$
9. $819 - 217 = 602$
10. $602 - 271 = 331$
11. $331 - 331 = 0$

Ans = 11